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Theory of Dimensional Randomness

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Abstract

There is still no scientific consensus on the existence of ontic randomness. Understanding it is especially important for physics, as the probabilistic nature of quantum mechanics is believed to be irreducible. This theory aims to explain how the existence of true randomness is possible for the referential of a non-omniscient observer. The theory is built on a toy model in two thought experiments. We propose the concept of invisible and inaccessible laws as explanation for the occurrence of unpredictability and states that it represents the physical reification of Gödel's incompleteness theorem. The article proposes that the geometry of spacetime is able to explain several physical phenomena in the set of dimensional random events. Among these are the event horizon in black holes, the hypersensitivity to initial conditions, and the Heisenberg's uncertainty principle. We develope the theorem of dimensional randomness for proving that a non-causal theory in a dimensionally incomplete space have correspondence with a causal theory in a dimensionally complete space. The theorem is used to question the fundamentally probabilistic nature of quantum mechanics through an implicit assumption that is made about the number of dimensions in spacetime for the development of Bell's Theorem.

Keywords: True Randomness, Heisenberg's Uncertainty Principle, Black Holes, Chaotic Systems, Bell's Theorem, String Theory

1 Introduction

For this paper, two definitions of randomness [1] are considered: the definition of subjective randomness (or epistemic, or pseudorandomness) by Demókritos and that of objective randomness (or ontic, or true randomness) by Epikuros. The first is randomness associated with ignorance of causation and a characteristic related to the lack of human acumen to determine the phenomena that influence the outcome of an event. The second definition, on the other hand, concerns randomness by causelessness, in which, even accurately measuring all initial conditions that influence an event, it is impossible to predict precisely the outcome. A pseudorandom event can become deterministic if the user is given more information, however, for a truly random event, there is no extra information that makes the event deterministic. The discovery of irreducibly random events is probably one of the most significant discoveries of the 20th century [2], but the existence of objective randomness is a hypothesis not yet confirmed by science [3]. The topic still involves much controversy, as it is very difficult to differentiate between a random or pseudorandom process given the complexity in proving the exhaustion of all possibilities that would influence the analysis of an event. In the field of quantum mechanics, the Copenhagen interpretation states that the collapse of the wave function is explained by the definitions of true randomness [4]. However, some important scientists disagree with this interpretation and philosophically take determinism as a principle, bealiving that quantum mechanics is an incomplete science that lacks a more fundamental theory. Because of this, the theory of hidden variables was formulated, which is based on realism and aims to describe how unobserved phenomena would lie behind the probabilistic nature of quantum mechanics. One of the biggest proponents of the Hidden Variables Theory was Albert Einstein (1879 - 1955). He wrote in a letter to Niels Bohr the famous sentence "God does not play dice". Through the EPR paradox [5], Einstein, Podolsky & Rosen tried to reduce to absurdity quantum mechanics by claiming that quantum entanglement implied the exchange of information violating the locality proposed by Special Relativity [6]. However, Bell demonstrated, via Bell's Inequality [7], that Hidden Variable Theories that assume both realism and locality are not consistent with the results of quantum mechanics. The Bell's CSHS test [8] is a way to test Bell's inequality and invalidate local realism. Based on these results, the Copenhagen interpretation gained strength and became widely accepted in the scientific community. The work in Bell's theorem was awarded with the Nobel Prize in Physics 2022. Through this work, we review the scope of Bell's Theorem to impose restrictions on the existence of hidden variables in accordance with the results of Quantum Mechanics, especially, reviewing the assumptions about what are hidden variables and what are the expected behaviors for these variables through the understanding of what represents dimensional randomness. The theory of Dimensional Randomenss is a geometric theory that proves dimensional randomness as a subset of events inside the set of true random events. Dimensional Randomness is a nomenclature proposed by this work for the uncertainty derived from the existence of invisible and inaccessible dimensions. From the results of our theory, interpretations can be created to describe objective randomness in several areas such as Statistics, Philosophy, Astronomy, Quantum Mechanics and Complex Systems. Algorithmically, a numerical series is random if the smallest algorithm capable of expressing it to a computer has the same number of bits of information as the series itself [9]. For the validation of our theory, we take the randomness described by our modeling and apply the distribution to an algorithmic analysis in order to check whether they fit the definition of randomness as described by Shannon's Information Theory [10] and Kolmogorov's complexity [11]. The non-comprehensibility of the sequences points out that the future of the systems is not computable by a deterministic turing machine. This theory can be considered an extensions for the algorithumic theory of information. In Kolgomorov's complexity, there is no difference between a pseudorandom sequence or a true random sequence. That is because, algorithmically, a pseudorandom sequence can mimic the absence of patterns. But a true random sequence differentiate by the absence of causality in a physical sense. Dimensional randomness represents a class of events in which causality cannot be determined by an observer due to irreducible physical limitations that will be explained in the thought experiments.

2 Thought Experiments

The Theory of Dimensional Randomness is formulated in two Thought Experiments on four abstract systems, not strictly concordant with the full understanding of physical laws. The systems are built as toy models, so that it can further a concise explanation to the occurrence of dimensional randomness as a particularization of true randomness. The basic premise we assume for the modeling of Thought Experiments is realism and the prohibition of mechanisms that create direct uncertainty or random behavior in any observed object. After developing the concepts and properties of the theory, we will check whether these extend validity to real physical systems.

2.1 Thought Experiment I

In the Thought Experiment we model a deterministic system, named s1, which will be used to generalize the concepts of the theory.

2.1.1 Description of s1

In the foundations of s1, there are 8 rules which are described below:

A System s1 is modeled on a space with 2 dimensions of space and 1 of time.
B The system s1 allows for circles and squares.
C In the initial settings, it is given that there are five circles identified by numbers from 1 to 5. Take a center c, with respect to c, each circle will have angular coordinates (ca) according to the formula: ca = circle number * 60º . The radial coordinate (cr) will be cr = circle number.

D The circles attract each other according to a vector v described by v = 6400 / d(1,2). Where d(1,2) represents the Euclidean distance between circles 1 and 2. At each instant of time, the circles displace in space a distance equal to the value of v in the direction and orientation of v. Squares also attract each other in the same way.
E If eight circles simultaneously reach a proximity threshold defined by a constant p, the eight circles are transformed into a square.
F Squares and circles repel each other according to a v2 vector described by v2 = -6400 / d(1,2). Where d(1,2) represents the Euclidean distance between the circle and the square. At each instant of time, the circles displace in space a distance equal to the value of v2 and in the direction and orientation of v2. The squares shift in space a distance equal to 1/8 the value of v2 in the direction of v2.
G Between discrete time instants, represented by a constant t, a new circle is generated at the mean coordinate of the existing circles at s1.
H Rules A through G are the most fundamental laws of the system s1 in the sense that there is no more fundamental rule that explains why they exist. The existence of the fundamental rules C,D,E,F and G are independent, that is, one rule could vary, without changing the other.

2.1.2 Inferences based in the simulation of s1

Based on the rules A-H, a computer simulation of s1 was made and we recorded 11 frames in the figure 1.

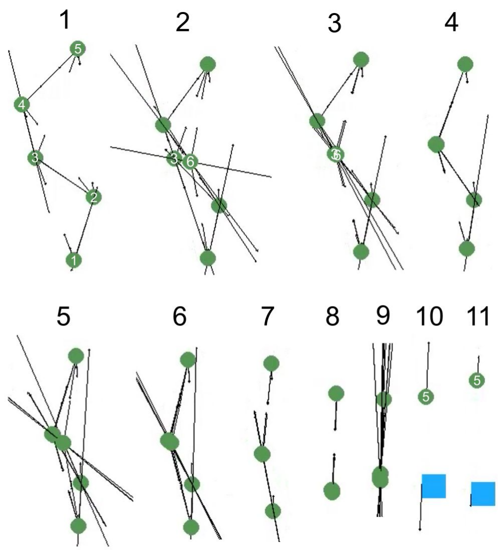

Fig. 1 s1 evolution

Frame 1 of figure 1 shows the s1 initial configuration with 5 green circles as described by rule C. The black arrows illustrate the attraction vectors between the circles. In the second frame, respecting G rule, observe the creation of a circle, numbered 6, at the mid-coordinate of the circles present in the system. In frame 3, circles 3 and 6 reach a limit of proximity in which it is difficult to visually differentiate the position of each circle. Between frames 4 and 9 in figure 1, rules D and G continue to exert an influence on the dynamics of the system. Especially in frame 10, the system reached a degree of specialization that allowed the manifestation of E rule for the creation of a square. In the next frame, it is already possible to observe how circle 5 changes its trajectory given the repulsion between circles and squares determined by F rule. The goal of the Thought Experiment in s1 is to demonstrate, by means of sentential calculus, how some phenomena are unpredictable for a fixed observer. To this end, we include an observer that will follow the evolution of the system s1 for a discrete duration of time. The properties of this observer are:

- I In principle, the observer does not know the A-H rules of the system s1.
- J The observer has access to all coordinates in space, however is limited to watch the passage of time between the instants represented by frame 1 and 4 of figure 1.
- K The observer can take measurements of any accessible variable in the system s1 with infinite precision.
- L The observer has an intellect capable of inferring, when observed, one or more equations to perfectly describe any occurrence in the system s1.
- M The observer witnesses the passage of time just as we do, being able to freeze frames 1 to 11 with absolute precision.

In this Thought Experiment, we wish to analyze whether the observer has the ability to perfectly predict the position of circle 5 in frame 11 of figure 1. To this end, we define a language of sentential calculus L composed of the following propositional variables: L = {A, B, C, D, E, F, G, vA, vB, vC, vD, vE, vF, vG}. The validity of sentential variables A, B, C, D, E, F, G means that the rules A, B, C, D, E, F, G of s1 are known by the observer. On the other hand, the sentential variables vA, vB, vC, vD, vE, vF, vG indicate, respectively, that the corresponding rules A, B, C, D, E, F, G were acted upon between the frames accessible for the observer. According to the properties of s1 and of the observer, we postulate the inferential basis in 3.

The base statements 1-7 mean that if the rules A-G act at an accessible instant, the observer deduces them due to the L property. Sentences 8-14 say that if the rules have not manifested at s1, the observer does not have the ability to deduce them, since the H rule of the system states that rules A,B,C ... G are at the most fundamental level of definition and are independent of C and G, so that knowing one rule does not contribute to understanding the other. Sentences 15-19 are findings based on the constraint defined by the observer property J and the observed events between frames 1 and 4 of the simulation. For proving unpredictability for the observer's referential, it is checked whether

Table 1 Inferential basis.

Number	Statement	Explanation
1	vA → A	L property from observer
2	vB → B	L property from observer
3	vC → C	L property from observer
4	vD → D	L property from observer
5	vE → E	L property from observer
6	vF → F	L property from observer
7	vG → G	L property from observer
8	¬vA → ¬A	H rule from s1
9	¬vB → ¬B	H rule from s1
10	¬vC → ¬C	H rule from s1
11	¬vD → ¬D	H rule from s1
12	¬vE → ¬E	H rule from s1
13	¬vF → ¬F	H rule from s1
14	¬vG → ¬G	H rule from s1
15	vC	J property from observer
16	vD	J property from observer
17	vG	J property from observer
18	¬vE	J property from observer
19	¬vF	J property from observer

the observer is able to predict exactly the position of circle 5 in frame 11 of figure 1. As pre-requirement, we postulate the following statement:

Table 2 Prediction postulate.

Number	Statement	Explanation
20	i ∧ (C ∧ D ∧ E ∧ F ∧ G) ⟺ p5	postulate

Statement 20 says that the observer predicts with absolute accuracy the position of circle 5 (p5) if and only if he determines with infinite accuracy the variables of the initial conditions (i) and knows all the rules C,D,E ... G that, in some of the frames 1 to 11 of figure 1, directly influenced the position of circle 5. By means of deductive mechanisms, the base sentences is developed in order to prove the validity of p5:

Table 3 Inferences.

Number	Statement	Explanation
21	i	K property from observer
22	C	Modus ponens, statements 3 ∧ 15 ∴ C
23	D	Modus ponens, statements 4 ∧ 16 ∴ D
24	G	Modus ponens, statements 7 ∧ 17 ∴ G
25	¬E	Modus ponens, ¬vE → ¬E, ¬vE ∴ ¬ E
26	¬F	Modus ponens, ¬vF → ¬F, ¬vF ∴ ¬ F
27	¬[i ∧ (C ∧ D ∧ E ∧ F ∧ G)]	¬F ∴ $\neg[i \wedge (C \wedge D \wedge E \wedge F \wedge G)]$
28	p5 → [i ∧ (C ∧ D ∧ E ∧ F ∧ G)]	Biconditional elimination of 20
29	¬p5	Modus Tollens, statements 27 ∧ 28 ∴ ¬ p5

Variable p5 is false since one of the requirements for p5 to be true is the simultaneous validity of E and F, but they have been respectively invalidated according to the union of the sentences 12, 18 and 13, 19 present in the basis. Because of this, there is unpredictability of the position of circle 5 from the observer's point of view. The existence of unpredictability, in the context of this Thought Experiment, is attributed to the specialization dynamics of the system. At s1, note that the fundamental rules have always existed in the logic of the system in an abstract way, but not all rules can immediately manifest themselves on the material plane of s1. For example, in order to form a square, it is necessary that there be eight circles simultaneously nearby, but in the initial configuration, there were only five circles and duly spaced at a distance greater than p. Such conditions are physical limitations that act as impediments to the formation of a square in the primitive stages of s1. Consequently, the knowledge accessible to the observer is not complete, since the properties inherent to squares cannot be studied by observation, inference and validation as required by the scientific method. Rules that cannot be studied at a given instant of time or location in space are named as invisible and inaccessible laws. They are a set of rules that potentially act on a system, but that by irreducible physical limitation, prevent inference and validation of their properties by a non-omniscient observer. Invisible and inaccessible laws can be represented as stocasthics diferential equations (SDE), but not all SDE represents invisible and inacessible laws. The invisible attribute indicates that the law is not observable in a given referential of time, and the inaccessible attribute states that the law is not inferable and validable under the same referential. The system s1 is modeled so that the rules E and F were invisible and inaccessible to any limited observer between the instants represented by frames 1 and 4 of the evolution of the system. Invisible and inaccessible laws represent the physical reification of Gödel's incompleteness theorem [12]. The existential cause of invisible and inaccessible laws for the referential of the observer is the invisibility and inaccessibility of the time dimension of s1 because if he could see through the time dimension, it would be possible to observe the information of all the frames of the system, and therefore vE and vF would be satisfiable. Taking, vE and vE $\rightarrow$ E; vF and vF $\rightarrow$ F, E and F can be deduced as true. Judging i, C, D, E, F, G to be simultaneously true, p5 also is true. However, It is not possible to generalize the results of Thought Experiment I to describe the existence of invisible and inaccessible laws or unpredictability in a real system, because the system s1 is a toy model and does not respect, with fidelity, physical laws. Rule G, for example, violates the principle of conservation of energy, since by creating new circles the total energy of the system increases. But this violation does not invalidate the inner results of the experiment for two reasons: 1st. The inference of unpredictability is not dependent on the increase of energy in the system, because for the rule of creating squares to manifest itself two prerequisites are necessary: a minimum number of circles in the system and proximity less than p between eight simultaneously nearby circles. The mere lack of proximity between the circles is sufficient to act as an impediment to the manifestation of G. 2nd. If the concept of energy was not specified in the fundamental rules of s1 this concept does not exist in s1 which does not make it inconsistent according to its own rules. Real physical systems are not used in experiment I as it is not possible to guarantee that we are omniscient to describe nature. Section 3 delves deeper to describe this requirement for the thought experiment. Thought Experiment I demonstrated that Laplace's Demon, proposed by Pierre Simon Laplace (1749-1827), is not valid for all moments of the evolution of s1 because the existence of invisible and inaccessible laws limits the intellect of any observer. To assert the validity of Laplace's Demon for any fundamentally deterministic system, it is necessary to prove that there are no invisible and inaccessible laws that affect the future states of the system. In the next step, Thought Experiment II is proposed and the manifestation of invisible and inaccessible laws describes unpredictability that fits a probability distribution.

2.2 Thought Experiment II

Thought Experiment II is an extension of Thought Experiment I. In this new experiment, the goal is to continue the exploration of the unpredictability observed in fundamentally deterministic systems. The focus is to explore how unpredictability can fit a probability distribution. Thought Experiment I showed that the position of circle 5 in frame 11 of figure 1 is unpredictable for an observer bounded in the referential represented by frames 1 and 4 of figure 1. However, an observer's prediction for the position of circle 5 can be adjusted if the observer follows the evolution of the system. The temporal evolution of s1 makes frames 5 to 11 visible and increases the scope of knowledge accessible to the observer, making it possible to resume the deterministic description for such a system if the observer's referential is extended. In this new experiment, we model a fundamentally deterministic system architecture such that the observer is forced to use probability distributions to describe certain phenomena.

2.2.1 Description of s2, s3 and s4

We define three parallel systems named s2, s3 and s4. All systems are abstract and described at the most fundamental level by deterministic rules not strictly in accordance with the understanding of physical laws.

The system s2 is described by the following rules:

A The system s2 is modeled over a two-dimensional vector space.
B The system s2 allows for circles and squares.
C In the initial configuration of the system s2 there are 8 circles. The location of these is determined by the polar coordinate system. Take the two-dimensional plane and a set of angular coordinates ca of 45º, 90º, 135º, 180º, 225º, 270º, 315º, and 360º for each of the circles, find the radial coordinate cr by means of the expression cr = ca / 45º .

D The circles attract each other according to a vector v described by v = 6400 / d(1,2). Where d(1,2) represents the Euclidean distance between circles 1 and 2. At each instant of time, the circles displace in space a distance equal to the value of v in the direction and orientation of v. Squares also attract each other in the same way.
E If eight circles simultaneously reach a proximity threshold defined by a constant p, the eight circles are transformed into a square.
F Squares and circles repel each other according to a v2 vector described by v2 = -6400 / d(1,2). Where d(1,2) represents the Euclidean distance between the circle and the square. At each instant of time, the circles displace in space a distance equal to the value of v2 and in the direction and orientation of v2. The squares shift in space a distance equal to 1/8 the value of v2 in the direction of v2.
G Between discrete time instants represented by t, a new circle is generated at the coordinate representing the simple arithmetic mean between the coordinates of the circles and squares existing in the systems s2 and s3.
H Between discrete instants of time represented by t, the oldest square in the system s2 is erased if there are more than 2 squares in the system.
I The variables and initial conditions of the system s2 are invisible and inaccessible to any observer external to s2 with the exception of communication with s4.

The system s3 is described by the following rules:

A The system s3 is modeled over a two-dimensional vector space.
B The system s3 allows for circles and squares.
C In the initial configuration of the system s3 there are 16 circles. The location of these is determined by the polar coordinate system. Take the two-dimensional plane and a set of angular coordinates ca of 22.5º, 45º, 67.5º ... 360º for each of the circles, find the radial coordinate cr by means of the expression cr = ca / 22.5º . It is defined that those closest to the arbitrary center are older, in chronological order, compared to the others.
D The circles repl each other according to a vector v described by v = - 6400 / d(1,2). Where d(1,2) represents the Euclidean distance between circles 1 and 2. At each instant of time, the circles displace in space a distance equal to the value of v in the direction and orientation of v. Squares also repel each other in the same way as circles.
E If 8 circles simultaneously reach a proximity threshold defined by a constant p, the 8 circles are transformed into a square.
F Squares and circles attract each other according to a v2 vector described by v2 = 6400 / d(1,2). Where d(1,2) represents the Euclidean distance between the circle and the square. At each instant of time, the circles displace in space a distance equal to the value of v2 and in the direction and orientation of v2. The squares shift in space a distance equal to 1/8 the value of v2 in the direction of v2.

G Between discrete instants of time represented by t, a new circle is generated at the coordinate representing the simple arithmetic mean between the coordinates of the circles and squares existing in the systems s2 and s3.
H Between discrete instants of time represented by t, the oldest circle in the system s3 is erased.
I The variables and initial conditions of the system s3 are invisible and inaccessible to any observer external to s3 with the exception of communication with s4.

The system s4 is described as follows:

A The system s4 receives the mean coordinate of the circles and squares in s2.
B The system s4 receives the mean coordinate of the circles and squares in s3.
C The system s4 calculates the simple arithmetic mean between averages of the coordinates of s2 and s3 and sends the result to s2 and s3.
D The system s4 does not allow for observers.

The characteristics of the observer are described below:

A The observer inhabits s3 but is external to s2 and s4.
B The observer can take measurements of any accessible variable in the system s3 with infinite precision.
C The observer has an intellect capable of inferring one or more equations to perfectly describe any occurrence of s3 with the exception of invisible and inaccessible laws.
D The observer is initially unaware of the fundamental rules of s2, s3 and s4.
E The observer witnesses the passage of time just as we do.

According to property A of the observer and property I of s2, the observer has physical limitations to deterministically infer the G rule of s3 since it presents direct dependence on variables of s2 and, according to property I of s2, is invisible and inaccessible to any observer external to the system. Because of this physical limitation, the G rule of s3 is an Invisible and Inaccessible Law to the observer referential. Based on the definitions presented for the 3 systems, it was implemented, in a computer simulation, the execution of s2, s3 and s4. The coordinates of the generated circles in s3 were registered for further analyses. It is evaluated how the interference of Invisible and Inaccessible Law G from s3 influences the predictions of an observer for the positions of circles. Three numerical experiments were carried out with the observer in order to verify the validity of the following sentences:

1. The accessible knowledge of s3 does not allow the observer to predict with absolute certainty the future position of the circles in the system.
2. The observer can use pattern recognition methods that remove uncertainty from the future position of the circles.
3. The possible inaccuracy in observer predictions recorded by experiments does not stem from floating point inaccuracy of our experiments.

Below, it is described the criteria for modeling the three experiments: In numerical experiment 1, disregarding the existence of invisible and inaccessible laws, it was used all the knowledge of systems s2, s3 and s4 to make predictions of the positions of the circles in space and time t + 1 of the system. The execution of this experiment, although not feasible from the point of view of the observer, aims to guarantee the validity of sentence 3 and the validity of the conclusions reached in numerical experiments 2 and 3. Next, numerical experiment 2 is performed. In this experiment, there is an observer capable of using only the knowledge of visible and accessible laws to make predictions. It is known that G rule of s3 is invisible and inaccessible to any observer, however, for the other rules, there is no restriction that prevents knowledge of the rule. Based on property C, the observer is able to deduce the laws A,B,C,D,E,F and H of the system s3 and use them to make predictions of the circle position in the instant t + 1. Numerical experiment 2 aims to validate sentences 1 and 2. In numerical experiment 3, as in numerical experiment 2, it was considered that the observer is able to use only the knowledge of visible and accessible laws to make predictions, but we added the ability to make assumptions about what would be the eventual invisible and inaccessible laws of the system. Possibly, the assumptions increase the assertiveness of the predictor method used. Experiment 3 re-evaluated sentences 1 and 2. Numerical experiments 1, 2 and 3 were performed on the same computer and under parity conditions of comparison.

2.2.2 Execution of numerical experiments in the system s3

In this section, we present the results of the application of the numerical experiments proposed in the mental experiment II. The first is the application Numerical Experiment 1, which applies a predictor that is capable of using the complete knowledge of the systems s2,s3 and s4. Considering a set of predictions made in the instant t before the appearance of a new circle, we recorded 75500 deviations of the expected and observed circle position in the time t + 1 of s3. In 75232 observations or 99.65% of the cases, there was no deviation and the difference between the actual value and the forecast was exactly equal to zero. In 268 observations or 0.35% of the cases, there was a deviation between prediction and observation. However, we do not attribute the existence of deviation in these observations to floating-point imprecision. The explanation for the existence of these deviations is a limitation of the prediction method that is taken in parallel with the execution of the system s2, s3 and s4. In cases where there was a deviation, the prediction considered, at time t, an old value of the mean that was present in the s4 system, but which, according to property A, received between times t and t + 1 an update of the mean of known coordinates of the system s2 and that caused the predictions to deviate from the real value at t + 1. In general, the prediction method that considers the use of all knowledge presented an average of the sum of deviations (x,y) of 0.00018 units of distance of s3 and 0.00017 of variance. In order for any recorded inaccuracy not to be attributed to the limitation of the numerical experiment, it is necessary that the deviations are significantly distant from those recorded in numerical experiment 1. We proceed with the application of numerical experiment 2 in which the observer is capable of using only the accessible knowledge to make predictions. We recorded, for the experiment, 500 thousand observations. Figure 2 shows the dispersion of deviations from the predictor method used. In 99.34%, there was a deviation between the prediction and the actual value. Compared with the reference predictor method, the difference between the number of perfect predictions is 98.9% in favor of the method used in numerical experiment 1. The average of deviations (x,y) added in this new experiment was 0.04881, an increase of 271 times in relation to that recorded in numerical experiment 1. Applying the Wilcoxon Signed Rank Test, we verified that the p-value is equal to 0, asserting that there are significant differences between the means.

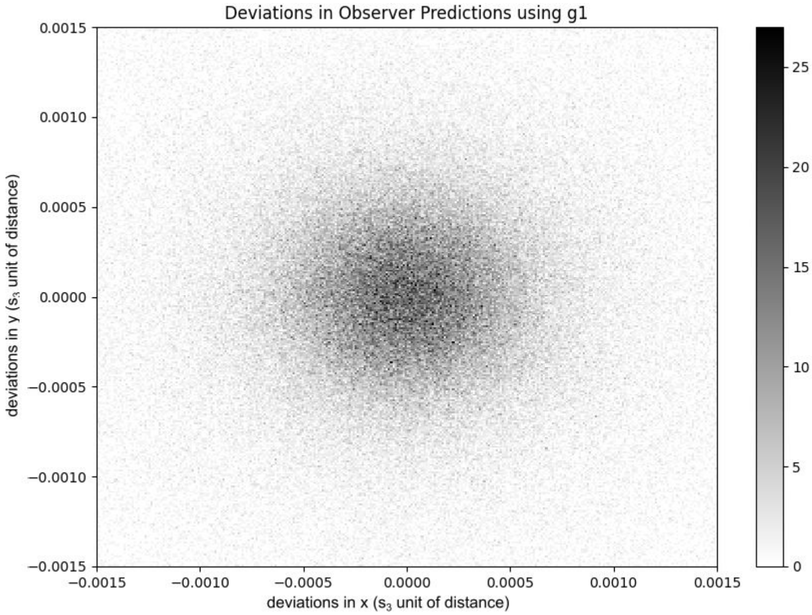

Fig. 2 Deviations of observer predictions in system simulation s2, s3 and s4.

Numerical experiment 2 validates sentence 1 and confirms that the use of knowledge of s3 accessible to the observer does not allow him to predict with absolute certainty the future position of the circles in the system.

In numerical experiment 2, all accessible knowledge to the observer is used to make predictions. However, we totally disregard assumptions that could be made about what the invisible and inaccessible law G of s3 would be. First, let's assume that the observer perfectly supposes the existence of s2 and also predicts that the average coordinate of s2 is influencing the coordinates of the circles created in s3. This assumption, although strictly correct, cannot be considered valid for this experiment, as it does not contribute to the improvement of the prediction method taken by the observer. After assuming the existence of s2, it would then be necessary to assume the initial conditions of the experiment, which include the position and number of circles in s2, the attraction between the circles, all the constants that influence the attraction and everything related to system s2. While this is not a valid assumption, we evaluate the invisible and inaccessible law G to estimate what assumption the observer could make to lessen the impact of ignoring G:

G: Between discrete instants of time represented by t, a new circle is generated at the coordinate representing the simple arithmetic mean between the coordinates of the circles and squares existing in the systems s2 and s3.

In the excerpt above, we verify that the invisible and inaccessible law G is dependent on a constant t and on the coordinates of the circles and squares in s3, information that is accessible to the observer. Based on this finding, we infer that, to make the predictions, the observer adds G2 rule to the set of known rules A-H:

G2: Between discrete instants of time represented by t, a new circle is generated at the coordinate representing the simple arithmetic mean between the coordinates of the circles and squares existing in the system s3.

We proceed with the execution of numerical experiment 3 which will use the accessible knowledge for the observer plus the assumption of G2. For this new experiment, we recorded 500 thousands observed deviations for the predictions and displayed, in figure 3, a heat map that represents the deviations after the adjustment assuming G2:

Visually, we observe that the deviations concentrated in figure 3 are lower than those recorded in figure 2. We verified that, in 99.9% of the cases, there was a difference between the predicted value and the actual value, a difference of 99.55% compared to the one recorded in numerical experiment 1. The average of the deviations (x,y) added in this new experiment was 0.05142, an increase of 300 times in relation to that registered in experiment 1. Although visually the results give the impression that the new method is a better predictor, the mean value of the deviations recorded in this new experiment was numerically higher than that recorded in experiment 2. To explain this apparent contradiction, we need to consider the distribution of values by interval. In numerical experiment 2, the range that contains 90% of the x-component data is approximately [-0.0327;0.0327]; the 99% confidence interval is [-0.526;0.526] and the 99.9% confidence interval is [-0.867;0.867]. After adjusting G2 in experiment 3, we found that the interval that contains 90% of the observations is smaller compared to before [-0.0205;0.0205], however the intervals of 99% and 99.9% are wider and with respective values [-0.664;0,664] and [-1.187;1.187]. According to the data, we found that the method that assumes G2 has a higher probability of approximating any prediction to the real value. In about 70% of the cases, the magnitude of the deviation in units of distance of s3 is smaller compared to the method that neglects G2, however, when the method that

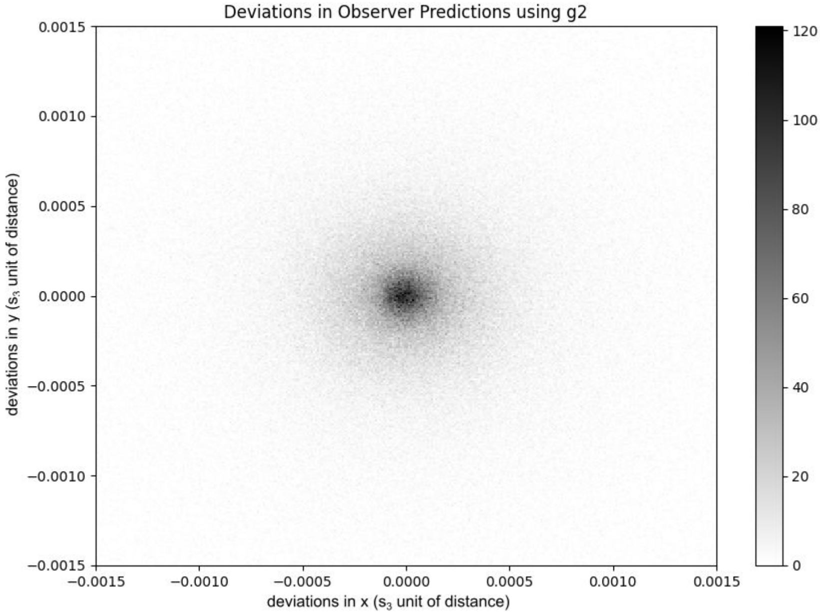

Fig. 3 Deviations of observer predictions in the simulation of s2, s3 and s4 after the assumption of G2.

assumes G2 deviates significantly from the real value, the deviations are more accentuated compared to the previous method. Because of this, we did not see improvements in the mean deviation of the new predictor. Using the Wilcoxon Signed Rank Test, we calculated that the p-value is equal to 0 and that there are significant differences between the prediction methods, with the previous method being 6% more accurate than this one. These results show that the assumption that could be made did not bring the observer closer to a deterministic description and reinforces the validity of sentence 1 that the use of accessible knowledge in the system does not allow predicting the future position of the circles. It is still necessary to judge the validity of sentence 2 which says that the observer may use a pattern recognition method that removes the uncertainty of the future position of the circles. To evaluate the previous sentence, we need to resort to definitions and numerical tests of algorithmic randomness. A numerical series is random if the smallest algorithm capable of expressing it to a computer has the same number of bits of information as the series itself [9]. Chaitin's statement can be measured by Kolmogorov Complexity. The Kolmogorov complexity K(x) of x is the length of a shorter string y such that x can be computed from y by a Fixed Universal Turing Machine [13]. However, K(x) is non computable [14]. Kolmogorov Complexity was extended to satisfy the strong invariance property and is related to the mutual information metric which satisfies the same property [15]. A robust test is proposed to check whether a sample distribution is random [16]. The test consists of

Table 4 Mutual Information between X1, X2 ... X12

MI	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
X1		.063	.018	.000	.000	.000	.000	.000	.001	.000	.000	.000
X2	.065		.076	.007	.001	.000	.001	.001	.000	.001	.000	.001
X3	.024	.078		.016	.001	.000	.001	.000	.000	.000	.000	.000
X4	.010	.011	.017		.001	.002	.001	.002	.000	.002	.001	.002
X5	.010	.007	.004	.002		.002	.001	.002	.002	.003	.003	.002
X6	.011	.007	.003	.002	.000		.001	.002	.000	.000	.002	.001
X7	.011	.008	.002	.001	.000	.000		.000	.000	.002	.003	.002
X8	.011	.007	.002	.002	.003	.002	.000		.001	.001	.002	.001
X9	.011	.008	.002	.002	.000	.001	.003	.001		.000	.000	.000
X10	.011	.008	.002	.001	.001	.001	.003	.003	.000		.001	.003
X11	.011	.008	.001	.000	.003	.001	.001	.000	.002	.001		.000
X12	.011	.009	.003	.004	.002	.000	.002	.002	.000	.001	.002	

taking a perfect random generator as a comparison measure. Then it is proposed to take the Kolmogorov-Smirnov test [17] to verify the hypothesis that a sample from the perfect generator and the tested sequence obey the same distribution. To perform the randomness test, it is taken the first 12 decimal places of the observed deviations in the numerical experiment 2 and is considered that each decimal place comes from a random variable Xn, in which n represents the position of the digit in the decimal places of the continuous number. The metric of mutual information is evaluated to check which digits are dependent. The variables Xn that have some degree of mutual dependence were disregarded from the randomness test because there is no guarantee that they respect the property of strong invariance, which must be satisfied when considering Kolmogorov complexity. The table 4 shows the calculated mutual information values between pairs of Xn.

The mutual information value means that by knowing one variable, you can be certain of a percentage of digits of information from the other variable. If the value of mutual information is 0, there is independence between the variables. Notice in table 4 that the values of dependence between pairs of variables Xn are generally low, the values are bounded between 0.000 and 0.076. We establish a criterion to interpret which values suggest dependence and independence between the variables. To this end, the chi-square test of independence [18] is taken at 95% confidence level. The values are recorded in table 5. The Bonferroni Correction [19] was applied to the p-values. In the cases where the letter D appears, the test result allowed us to reject the null hypothesis and confirm the dependence between the variables. In the cases represented by the letter I, the test did not reject the null hypothesis and it is not possible to detect dependence between the two variables.

Notice, in table 5, that between the variables X1 and X5, the test detected that there is dependence, while, beyond the variables X5, no dependence was detected. Based on this, we proceed to apply the randomness test proposed by Shen (2020) on the variables X1, X2 ... X12. The random module of python [20] is taken as a reference generator. Then, 7 samples of 500000 independent and uniformly distributed integers are generated in the interval between 0 and

Table 5 Chi-square test of independence between X1, X2 ... X12

RV	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
X1		D	D	D	I	I	I	I	I	I	I	I
X2	D		D	D	D	I	I	I	I	I	I	I
X3	D	D		D	D	I	I	I	I	I	I	I
X4	D	D	D		I	I	I	I	I	I	I	I
X5	I	D	D	I		I	I	I	I	I	I	I
X6	I	I	I	I	I		I	I	I	I	I	I
X7	I	I	I	I	I	I		I	I	I	I	I
X8	I	I	I	I	I	I	I		I	I	I	I
X9	I	I	I	I	I	I	I	I		I	I	I
X10	I	I	I	I	I	I	I	I	I		I	I
X11	I	I	I	I	I	I	I	I	I	I		I
X12	I	I	I	I	I	I	I	I	I	I	I	

Table 6 Metrics of Kolmogorov-Smirnov Test between randint and X1, X2 ... X12

RV	D Statistic	p-value	adjusted p-value
X1	0.852253	0	0
X2	0.69383	0	0
X3	0.38587	0	0
X4	0.10114	0	0
X5	0.01003	<<0.01	<<0.01
X6	0.00074	0.999	1
X7	0.00208	0.232	1
X8	0.00099	0.968	1
X9	0.00150	0.629	1
X10	0.00219	0.183	1
X11	0.00228	0.148	1
X12	0.00169	0.471	1

9 by means of the randint function. To each of these samples the Kolmogorov-Smirnov test is applied to verify, at 95% confidence, that the sampling of values of Xn and the sampling of values of the random module follows the same distribution. For the variables X5, X6 ... X12, the test failed to reject the null hypothesis and suggests that Xn is distributed like a discrete uniform distribution in the interval [0,9]. See the metrics of Kolmogorov-Smirnov test in the table 6:

Based on the results of table 4 and 6, the combination of variables X6, X7 ... X12 did not fail any of the tests. We ensure that knowledge of a variable Xn does not decrease the uncertainty of a variable Xn+c, where $n \geq 6$. Randomness test includes the evaluation of serial correlation metric. The serial correlation measures the relationship between a variable and a lagged version of itself. It is a mathematical tool to detect periodicity in the data. The serial correlation measure has already been used for randomness testing, with emphasis on a serial correlation permutation test proposed by Wald & Wolfowitz [21]. The Rh serial correlation is measured for the RV X1, X2 $\ldots$ X12. In table 7, It is shown the maximum Rh for the RV varying the lag parameter

Table 7 Maximum serial correlation varying the h parameter in the range [1,500000]

RV	max(R1,R2,. . . ,R50250)	RV	max(R1,R2,. . . ,R50250)
X1	0.270789	X7	0.007012
X2	0.506885	X8	0.005890
X3	0.472550	X9	0.006338
X4	0.158905	X10	0.005800
X5	0.005646	X11	0.005709
X6	0.006055	X12	0.006359

h in the range of 1 to 500000. We evaluate the maximum Rh because it represents the worst case of periodicity observed between any two digits in the lag interval [1,500000].

To interpret the values of table 7, we ran 20000 independent samples of 500000 observations of the random reference generator. And estimated the 99% confidence interval to be [0.00509; 0.00706], the variables X1, X2 . . . X4 failed the serial correlation test because the values are significantly distant from the confidence interval. Considering the random variables X5, X7, X8 . . . X12, the maximum values of Rh are inside the 99% confidence interval. Based on these metrics, the variables X5, X7, X8 . . . X12 did not present regularities that significantly differentiate them from a random reference generator in the analyzed interval. The metrics point out that starting from the sixth decimal place, there is no evidence of patterns in the variables and the system behaves like a real random system. It is not ensured that a pattern may not occur in a future state of the system, actually, The Poincaré recurrence theorem [22] states that after a very long period, a system that preserves its volume returns to a very close scenario as in the initial conditions. But while the pattern does not appear, the observer cannot use a pattern recognition algorithm to take a deterministic description of s3. So there is a moment in which unpredictability is irreducible in the system. Actually, we expect that a pattern may never occur since the experiment is inspired by Brownian Motion [23] and the G rule does not preserve the system volume. The distribution in figure 2 is extremely relevant to this experiment, because even if it is possible for us to explain the causes for the deviations of predictions, due to the invisibility and inaccessibility of G for the observer's referential in s3, there is not an existential cause for the imprecision observed. For any non-omniscient observer in s3, the complete description of G is inaccessible, with modified G being the only possible description for G:

G Between discrete instants of time represented by t, a new circle is generated at the coordinate representing the simple arithmetic mean of the circles and squares present in s3 . To the value , add a random offset (x,y) sampled from f3(x,y), in which f3(x,y) represents the probability density function of the deviation from .

Because modified G contain a probability function, there are some deterministic systems in which randomness is irreducible for its description. The definition of objective randomness describes a random event that is associated with the absence of cause. Under an observer's frame of reference, objective randomness is described as a relationship between the spacetime topology and the physical laws of a system. The actuation of invisible and inaccessible laws fits the definitions of objective randomness, because it is not possible, under the observer's referential, to distinguish the class of events that have a fundamentally unknown cause from the class of events that are fundamentally absent of cause. In section 3, we provide a geometric interpretation that deepens the referential view of objective randomness.

Code Availability: The python code used to run the Thought Experiments I and II with the instructions to replicate the experiments are available on Github[1].

3 Geometric Interpretation of Objective Randomness

Thought Experiments I and II is generalized to a geometric interpretation which will be extended to describe true randomness in real physical systems. In the thought experiment I there was an asymmetry in the way we and the observer use deductive logic. Because we are the creators of the s1 system, we can know all the fundamental rules of the system and use the closed world assumption[24] to make inferences. The closed world assumption says that a true sentence is also known to be true in the logical system. The possibility of using the closed world assumption gave us the possibility of exactly simulating the future of the system, since all true facts are known to be true. Building precisely the simulation of the future gave us one more degree of freedom to move around the system, giving us the ability to travel into the future and analyze what facts in the system's past were impossible to predict by an observer who was using the world open assumption [25]. Therefore, using the closed world assumption allowed us to move through a dimensionally complete space, while the observer was contained in a dimensionally incomplete space, making him a lower observer of the system in relation to us.

Figure 4 exhibits the block-universe representation of Thought Experiment I. The toy model used this representation instead of Einstein's theory of 4D spacetime because the block universe provides extra capabilities for the 2D observer to describe the universe. The first, represented by the J property of the observer, is the ability to measure the state of all space variables at a given instant. But according to the Special Theory of Relativity [6] that capability is not feasible because the light speed c is an upper limit for the propagation of information. The second, represented by M property of the observer, is the capability to despise the time dilation proposed by the General Theory of Relativity. We want the observer to have the least physical limitations to describe the universe deterministically, being any eventual physical limitation only inherited by our proposed toy model. In figure 4, the two spatial dimensions x and y represent the coordinates at which the material entities of s1

[1]https://github.com/LucasLopesSI/ThoughtExperiments

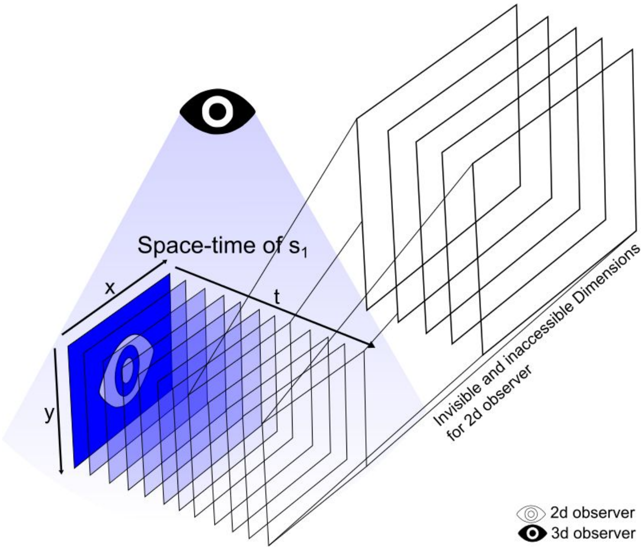

Fig. 4 s1 geometry in Thought Experiment I.

can take at a given instant. The time dimension t, on its turn, describes the stack of spatial frames of the system, and it is possible to describe the position of any object in spacetime assuming that there is visibility and accessibility of the time dimension. Two observers are also represented in the figure. The first observer, represented by a white eye, is limited to 2 dimensions and is therefore dimensionally inferior to the second. The lower observer inhabits s1 and is able to measure the variables present in the system at a given instant with infinite precision. The second observer, represented by the black eye, is dimensionally superior to the first since it can visualize the spacetime of s1 in 3 dimensions. In figure 4, the dimensions visible and accessible to the lower observer are represented by the blue color and the invisible and inaccessible dimensions are transparent. Unpredictability for an observer occurs when he encounters limitations that prevent the visibility and accessibility of a dimension. In this sense, he does not have available the set of information that would allow him to reconstruct the rules of the system.

In Thought Experiment I, it was not possible to derive a probability distribution that describes unpredictability. This is because the invisible and inaccessible dimensions are connected to the lower observer through the time component, and from the specialization of the system, the invisible and inaccessible law acts. For a sufficiently intelligent observer, it is possible to deduce the law that was previously invisible and inaccessible to avoid unpredictability.

Thought Experiment II is designed in order to explain how unpredictability fits a probability distribution. In this new experiment, a system architecture with extra spatial dimensions is proposed using the parallelism of s2 and s3. Moreover, there is an additional system s4 that is responsible for the communication between s2 and s3. A two-dimensional observer is embedded in

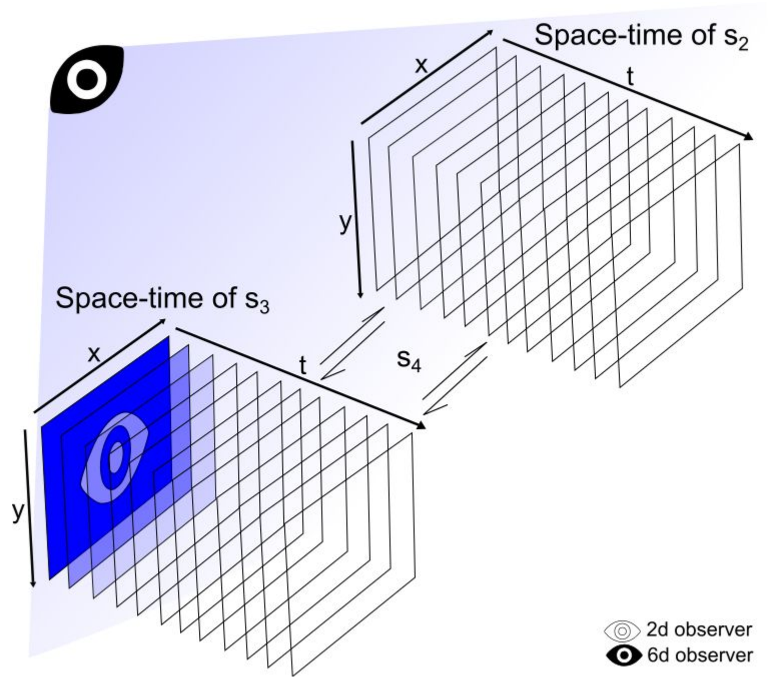

Fig. 5 Geometry of s2,s3 and s4 in Thought Experiment II.

the spacetime of s3. We prove that to describe the position of circles in s3, the lower observer must use a probability distribution and that the numerical imprecision obey the algorithmic definition of randomness. Figure 5 shows the architecture of the Thought Experiment II.

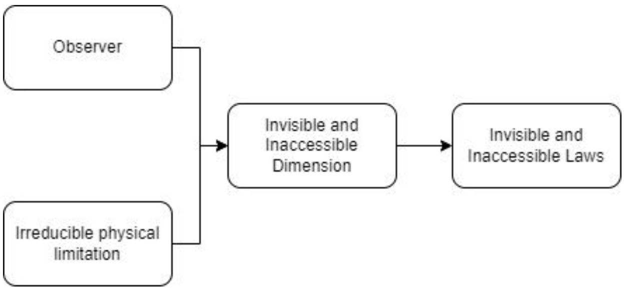

Fig. 6 General mechanism for the existence of objective randomness according to Theory of Dimensional Randomness.

Figure 6 describes the mechanism that was able to give rise to objective randomness in thought experiments I and II. Initially, there is the existence of an observer and one or more physical limitations. In thought experiment I, the physical limitations for the observer is the possibility of living only in the present and for a limited duration between frames 1 and 4. Furthermore, the system was not specialized enough for the observer to theoretically or experimentally predict the repulsion rule between squares and circles. In thought experiment II, the observer is only able to live in the s3 system, whereas the systems s2 and s4 are parallel to its existence and exchange information in a way that the observer cannot observe or infer. Physical limitations vary according to the type of system in which the observer is inserted. Using real systems, we will address some physical limitations in section 5 of this article. The observer's association with irreducible physical limitations is the cause for the existence of invisible and inaccessible dimensions. And the laws that act in these dimensions, inherit the invisible and inaccessible attributes, being necessarily described by probability theory. Thought experiments I and II were modeled on hypothetical and non-physical systems, because in this way we guarantee omniscience of the events that happen in the system. As superior observers, the Closed-World Assumption is used to distinguish the class of events with a fundamentally unknown cause from the class of events with an unknown cause because of the observer's lack of acumen. Using real physical systems, we must assume the Open-World Assumptions since we are 3D observers immersed in a 4D universe, then the distinction between the two classes of events becomes fuzzy.

4 Theorem of Dimensional Randomness

Based on the insights and experimental results of thought experiments I and II, we propose the conjecture of dimensional randomness.

Conjecture of Dimensional Randomness:

An accessible space dimensionally inferior[2] to the dimensionally complete space[3] implies in random events described by F, in which F represents a probability distribution.

Hypothesis 1:

There is a dimensionally random probability distribution F if 1.1 or 1.2 hypothesis are true:

$$f(x) = \begin{cases} \exists F, & \text{if } [(\mathbb{R}^{t1,n}_{incomplete} \wedge \mathbb{R}^{t2,m}_{complete} \mid t1 + n < t2 + m) \vee (\mathbb{R}^{t1,n}_{incomplete} \neq \mathbb{R}^{t1,n}_{complete})] \end{cases} \tag{1}$$

Hypothesis 1.1:

The Minkowski space denotes that the number of dimensions (t1+n) in observable space is irreducibly smaller than the number of dimensions in the dimensionally complete space (t2+m).

[2]dimensionally incomplete space is a space which contains only the dimensions accessible to an inferior observer and do not comprehend invisible and inaccessible dimensions.

[3]Dimensionally complete space is a space which has all the degress of fredoom acessible, including invisible and inacessible dimensions for any inferior observer.

$$\mathbb{R}^{t1,n}_{incomplete} \wedge \mathbb{R}^{t2,m}_{complete} \mid t1 + n < t2 + m \tag{2}$$

Proof of the conjecture for hypothesis 1.1:
The equivalence bellow relates the perspective of the mathematical relationships mapped in the dimensionally incomplete space to the functions belonging to the dimensionally complete space:

$$\begin{aligned} f1(x, y, ..., z) &\equiv f2(x, y, ..., z, ..., z1) \\ f1(x, y, ..., z) &\equiv f2(x, y, ..., z, ..., z2) \\ &... \\ f1(x, y, ..., z) &\equiv f2(x, y, ..., z, ..., zn) \end{aligned} \tag{3}$$

In 3, $f1(x, y, ..., z)$ represents a mathematical relationship that receives as input the observables (x,y, ..., z) for all degrees of fredoom in the dimensionally incomplete space. The mathematical relationship $f1$ outputs the result for an observed event in the system. On the other hand, $f2(x, y, ..., z, ..., z1)$ is a function belonging to the dimensionally complete space that receives as input the observables (x,y, ..., z, ..., z1) for all the degrees of fredoom in the dimensionally complete space. Because $f2$ is a function in the space, we state that the dimensionally complete space has realism. For every function, each element of the domain can only have one correspondence in the co-domain set. The equivalence in 3 means that the observer inhabits and observe the results from the functions in the dimensionally complete space, but can only formalize and predict the process results by means of mathematical relationships with less degrees of freedom compared to the dimensionally complete space. In 4, We relate the solutions of the functions in the dimensionally complete space to a set of elements ω_1, ω_2, ..., ω_n

$$\begin{aligned} f2(x, y, ..., z, ..., z1) &= \omega_1 \\ f2(x, y, ..., z, ..., z2) &= \omega_2 \\ &\cdots \\ f2(x, y, ..., z, ..., zn) &= \omega_n \end{aligned} \tag{4}$$

We arbitrarily choose a configuration of input parameters $(x, y, ..., z...z1), (x, y, ..., z...z2)$, ..., $(x, y, ..., z...zn)$ that produce strictly different solutions, so that:

$$\begin{gathered} f2(x, y, ..., z, ..., zi) \neq f2(x, y, ..., z, ..., zj), \\ \forall i \in \{1, 2, 3, ..., n\} \wedge j \in \{1, 2, 3, ..., n\} \wedge i \neq j \end{gathered} \tag{5}$$

The 5 statement may seem contradictory to the 3 statement because if we apply the transitive rule in 3, we get to $f2(x, y, ..., z, ..., z1) \equiv f2(x, y, ..., z, ..., z2) \equiv, ..., \equiv f2(x, y, ..., z, ..., zn)$. But if we replace a random element in 5 by its equivalent term, we obtain the contradiction $f2(x, y, ..., z, ..., z1) \neq f2(x, y, ..., z, ..., z1)$, which deny the initial assumption that f2(x,y, ..., z, ..., z1) is a function in the dimensionally complete space. But the reasoning that led to this contradiction is not correct because it is mixing up two disjoint sets of statements as shown in figure 7.

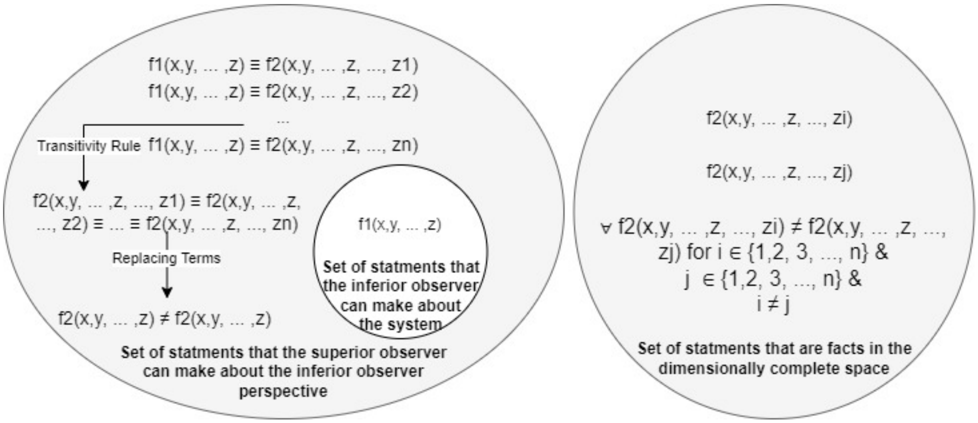

Fig. 7 Disjoint set of statements.

In figure 7, note that after applying the transitivity rule, the new statement keeps belonging to set of statement that the superior observer can make about the inferior observer perspective. But the statements from this set can not be used to change the statements in the set that contains the facts from the dimensionally complete space. This occurs because the statements of both sets deal with different domains of logic. While the set on the right deals with the facts in the space with respect to the closed world assumption, the set on the left contains the statements that a superior observer uses to describe the perspective of an inferior observer in relation to a system through the open world assumption, but which do not have the capacity to influence the fundamental rules of a system that by premise is realist. Formally, the set of facts in the dimensionally complete space can monotonically expand through an axiomatic basis closed in the set. But, the set of statements that the superior observer can make about the inferior observer can monotonically expand using both open and closed world assumption in the following cases: 1. the statements in the set of facts in the dimensionally complete space because the superior observer uses the closed world assumption and knows all facts of the system; 2. the statements in the set of statements that the inferior observer can make, because the superior observer can deduce what is the perspective of an inferior observer; 3. The inner statements of the set. After explaning this inference restriction, we follow the proof expanding the set of statements using the perspective of the superior observer.

We replace the functions from the dimensionally complete state of 4 by its equivalent mathematical relationships for the inferior observer of 3, we get to the following statement:

$$\begin{aligned} f1(x,y,...,z) &= \omega_1 \\ f1(x,y,...,z) &= \omega_2 \\ &\dots \\ f1(x,y,...,z) &= \omega_n, \end{aligned} \tag{6}$$

In 6, we can be sure that, by the perspective of the inferior observer, the dimensionally incomplete space has not realism. That is because 6 is relating a single mathematical relationship $f1$ with a predefined configuration of input parameters (x,y, ..., z) to a set of results { ω_1, ω_2, ..., ω_n}. By statement 5, we know that each ω value is different from each other, which means that $f1$ is a mathematical relationship in which the elements from the domain set can have more than one correspondence in the codomain set, therefore not respecting the definiton of realism. A random event is a relationship in which the same initial conditions (x,y, ... ,z) can originate different output states $\omega_i \neq \omega_j$. We note that f1(x,y, ... ,z) respects this definition, thus representing a random event F under the observer's frame limited to the dimensionally incomplete space $\mathbb{R}^{t1,n}_{incomplete}$.

Theorem 1.1:

A dimensionally incomplete space has not realism and implies in dimensional random events described by F. In which F is a probability density function.

$$[\mathbb{R}^{t1,n}_{incomplete} \wedge \mathbb{R}^{t2,m}_{complete} \mid t1 + n < t2 + m] \implies F \tag{7}$$

Hypothesis 1.2:

$$\mathbb{R}^{t1,n}_{incomplete} \neq \mathbb{R}^{t1,n}_{complete} \tag{8}$$

Hypothesis 1.2 says that there is a dimensionally complete space which has the same numbers of dimensions as the dimensionally incomplete space, but which are strictly different.

Proof of the conjecture for hypothesis 1.2:

$$\mathbb{R}^{t1,n}_{incomplete} \neq \mathbb{R}^{t1,n}_{complete} \equiv \exists[\alpha \in \mathbb{R}^{t1,n}_{complete} \wedge \alpha \notin \mathbb{R}^{t1,n}_{incomplete}] \tag{9}$$

9 says that the difference between the spaces is equivalent to defining that there is an α that belongs to the dimensionally complete space, but does not belong to the dimensionally incomplete space. The equivalence can be made, as the dimensionally incomplete space cannot be larger than the dimensionally complete space. As the spaces are different, the only remaining alternative is $\mathbb{R}^{t1,n}_{incomplete} \subset \mathbb{R}^{t1,n}_{complete}$

$$\neg f \vee \neg \alpha \implies \neg f(\alpha) \tag{10}$$

We postulate that to predict the outcome of an event determined by a hypothetical function $f(\alpha)$ it is necessary to know the function f and the input parameter α.

$$\neg f(\alpha), 9 \wedge 10 \tag{11}$$

11 states that $f(\alpha)$ is not true because 9 states that α does not belong to the dimensionally incomplete space.

$$\alpha \approx \alpha_1 + \epsilon, \alpha \in \mathbb{R}^{t1,n}_{complete} \wedge \alpha_1 \in \mathbb{R}^{t1,n}_{incomplete} \wedge \epsilon \in \mathbb{R}^{t1,n}_{incomplete} \tag{12}$$

But, the α may be approximated by an $\alpha_1 \in \mathbb{R}^{t1,n}_{incomplete}$ plus an estimated error ϵ.

$$\epsilon \sim F_x(x) \tag{13}$$

The ϵ is distributed like a probability density function F.

Theorem 1.2:

$$\mathbb{R}^{t1,n}_{incomplete} \neq \mathbb{R}^{t1,n}_{complete} \implies f(\alpha_1 + F) \tag{14}$$

A dimensionally complete space that has its dimensions partially inaccessible implies in an error distributed like F for the inaccessible states.

5 Applications of the Theory of Dimensional Randomness

This section extends the application of the Theory of Dimensional Randomness to explain phenomena and constraints observed in subfields of Mathematics, Physics, and Computing.

5.1 Why do conventional computers use pseudorandom number generators (PRNG)?

According to the Theory of Dimensional Randomness, the requirement for the creation of an objectively random function is the invisibility and inaccessibility of one or more dimensions in space for the referential of a non-omniscient observer and associated with the operation of at least one Invisible and Inaccessible Law. The first obstacle to creating objective randomness in modern computers occurs because the architecture and models of computation are specified. In this way, computers are implemented based on models controlled by humans, making us able to use the closed world assumption to deny the existence of objective randomness. We even understand the most fundamental level of a computer's information and, as a result, become an omniscient observer of the computations that take place. However, in physics, the process is opposite. Theories and predictions are modeled so that they fit the observations of the real world. The open world assumption must be taken, implying that ignorance of a certain phenomenon is not sufficient for the denial of it. The second drawback in using conventional computation to generate objective randomness occurs because the computations produced by a machine are in the same spatial dimensions that are inhabited and accessible to us. Because of this, there is no capability to create physical constraints on the operation of invisible and inaccessible laws. To deal with this problem, we strive to create invisibility of the random root, and complexification of the function used for random number generation, however invisibility is a weaker property compared to inaccessibility, allowing computations of a pseudorandom function to be reduced to deterministic computations by an observer that breaks the invisibility of the process.

5.2 Decay of Deterministic Equations

The operation of invisible and inaccessible laws that are not considered when writing an equation adds randomness to the future predictions of a system. The evolution of this phenomenon can occur in a smooth manner, so that predictions lose accuracy over time. We call this process the Decay of Deterministic Equations, which is defined as the transition from a deterministic equation to a random function. The transition occurs from the manifestation of invisible and inaccessible laws in the observational results of the system. Figure 8 represents the decay process for a deterministic equation.

In figure 8, the upper left image demonstrates the predictions made by a deterministic equation. Notice that there is a collapse of the function at a well-defined peak. This is because the equation behaves deterministically at a given time, and there is certainty of an outcome. As new invisible and inaccessible laws manifest in the system, the quality of the predictions are reduced, and can be observed through the emergence of a more curvilinear and smoother mountain in the plane. The complete decay of a deterministic equation occurs after the predictions made are comparable to predictions based on a uniform distribution over the sample space. You can see the result of the complete decay of a deterministic equation in the lower right corner of the figure. Take the system s1 formed only by circles and squares and an equation which aims to describe the trajectories of the circles, but which considers only the attraction between the circles. Suppose that this system has a large number of circles and that at a hypothetical instant the first emergence of a square is observed from the fusion of 8 circles. Taking that the repulsion between squares and circles is weak over long distances to the point of being negligible, we can state that

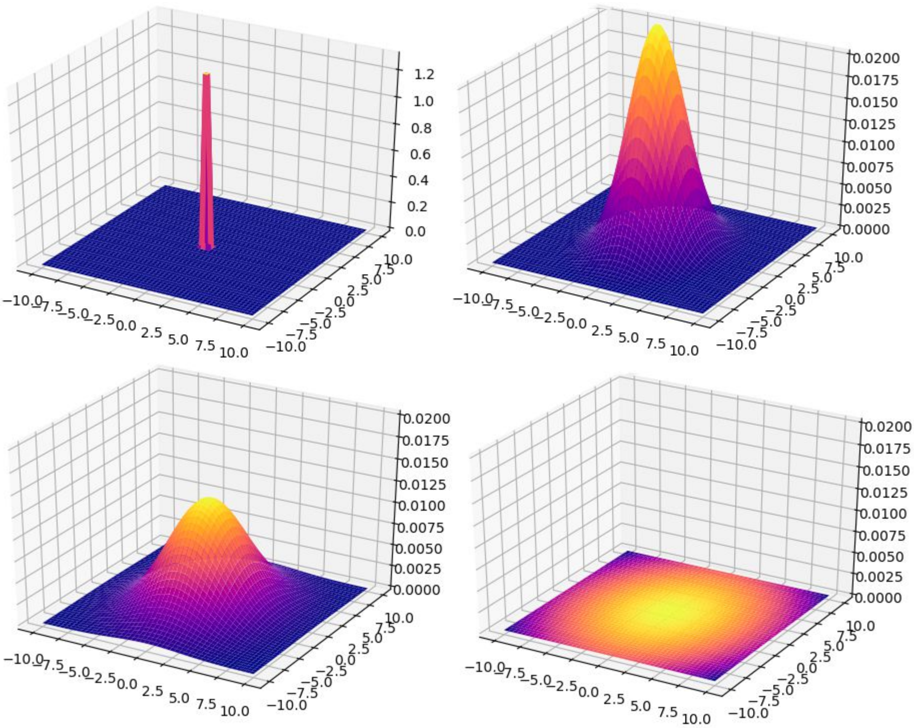

Fig. 8 Smooth decay of deterministic equations.

the initially written equation can describe with high accuracy the trajectory of circles that are not close to squares. However, as new squares are added to the system, the equation is less assertive in describing the trajectories of circles. If the predictions of the equation are as bad as uniformly random estimations in the sample space, the equation is said to have undergone complete decay to a random function. To avoid the process of decay and aging of the equation, updates and revisions are necessary according to the periodicity of the invisible and inaccessible laws acting on the system.

5.3 Objective Random and Physical Phenomena

In subsections 5.3.1 to 5.3.5, we describe 5 examples of physical phenomena that fit the definitions of objective randomness under the definitions of theorems 1.1 and 1.2 provided in section 4 of this paper.

5.3.1 Event Horizon

The speed of light in a vacuum is a constant c equal to 299,792,458 m/s and invariant to the referential [6]. It is not possible to exceed the speed of light, because it represents the speed of propagation of spacetime itself. Schwarzschild provided the first description of what a black hole would be [26] according to the solutions of Einstein's equations of General Theory of Relativity [27]. Finkelstein interpreted black holes as a region of space in which it is not possible to escape, because if the mass of a given body is compressed to a radius less than the Schwarzschild radius, in order for bodies entering the Schwarzschild radius to escape the gravitational influence of the black hole, they must have an escape velocity greater than the speed of light in vacuum [28]. However, it is not even possible for light to exceed this speed. Thus, the information contained in a black hole is invisible to observers outside it. The study of events within the event horizon are speculative. Generalizing the results of the field equation of the theory of General Relativity, it is described that inside a black hole there is a gravitational singularity, a point in spacetime at which the mass and curvature of spacetime are infinite. However, these results cannot be experimentally proven because black holes cannot be accessed in order to compare or validate the results directly or indirectly. Unless black holes propagate the internal content to the outer environment, the definition proposed by this article for objective randomness is still valid. New scientific discoveries may demonstrate that information inside a black hole can be propagated to the external world, for example, the existence of an Einstein-Rosen bridge at the event horizon. But, black holes lose mass through Hawking Radiation and during this process convert a pure state into a thermal state, losing the information contained in the process [29]. Taking current physical definitions, black holes are described as invisible and inaccessible regions of the spatial dimensions. More Formally, we can state that $\exists[\alpha \in \mathbb{R}^{t1,n}_{complete} \wedge \alpha \notin \mathbb{R}^{t1,n}_{incomplete}]$ is true given the definition of Black Holes. In this sense, within the event horizon, invisible and inaccessible laws can act only under the specific circumstances that black holes provide, so that the knowledge formulated on the observations of the visible and accessible part is not generalizable, suffering, in this way, random decay of the predictions made by deterministic equations. The results of the generalization of general relativity for the description of black holes present indications of random decay, as several physical quantities cease to make sense and assume infinite values. According to the current description of physics and using theorem 1.2, we demonstrate that the event horizon of black holes gives rise to dimensional random event. It is believed that a quantum-gravity theory explains the events at the event horizon of black holes, but the existence of such a theory that unifies the four fundamental forces does not rule out the hypothesis that, exclusively, at the event horizon a fifth, sixth or nth force may act and that it is invisible and inaccessible to our referential. The definition of Objective Randomness remains valid unless new scientific facts prove the possibility of breaking the invisibility and inaccessibility of the event.

5.3.2 Butterfly Effect

The flapping of a butterfly's wings can cause a tornado in Texas [30]. The interpretation for this phenomenon is the sensitivity to the initial conditions measured for a system, in which small inaccuracies can trigger chaotic effects that are amplified through the propagation of system states. The analogy, dubbed the butterfly effect, was used to explain why weather forecasts deviated after a day or two to degrees of uncertainty beyond what was expected. Theoretically, to make predictions with high assertiveness, it is necessary to collect data with arbitrarily defined precision, in addition to complete knowledge of the laws that govern the dynamical system. The level of precision for the measurements of an experiment is limited according to the quality and advancement of technological instruments, which makes the unpredictability observed for chaotic systems conform to the definition of Subjective Randomness. The measurement capability of the instruments applies to finite values. For measuring variables belonging to the set of irrational numbers $\mathbb{I}$ there are limitations, because, by definition, irrational numbers cannot be represented as fractions of two integers. It's proved that the decimal expansion of irrational numbers is infinite and non-periodic [31]. Because any measurement has a finite decimal expansion, any variable in which its value belong to the set of irrational numbers cannot be precisely measured. Formally, $\forall\alpha \in \mathbb{I}$, then $\alpha \in \mathbb{R}^{t1,n}_{complete}$, but $\alpha \notin \mathbb{R}^{t1,n}_{incomplete}$. Given 9, we deduce that $\mathbb{R}^{t1,n}_{incomplete} \neq \mathbb{R}^{t1,n}_{complete}$. By theorem 1.2, the prediction of events in which the initial conditions are described by irrational numbers is dimensionally random.

5.3.3 Heisenberg's Uncertainty Principle

The Heisenberg's Uncertainty Principle [32] defines that it is impossible to simultaneously determine the position and momentum of a particle with arbitrary precision for both variables according to the expression:

$$\Delta p.\Delta x \geq h/(4\pi) \tag{15}$$

Where p represents the uncertainty of the position x the momentum uncertainty and h the planck constant. The mathematical inequality expressed above indicates that if the uncertainty of one of the variables in the pair (p, x) decreases, the other variable in the pair must increase its value for the expression to hold true. The Heisenberg uncertainty principle represents a physical limitation that prevents one from being able to measure with absolute certainty, simultaneously, the momentum and the position of a particle. Due to this limitation, we point out that the reconstruction of particle information in the 3 dimensions of space is partial, making these dimensions partially invisible and inaccessible. We rewrite the The Heisenberg's uncertainty principle in the following terms:

$$f(x) = \begin{cases} p \in \mathbb{R}^{t1,n}_{incomplete} \implies x \notin \mathbb{R}^{t1,n}_{incomplete} \\ x \in \mathbb{R}^{t1,n}_{incomplete} \implies p \notin \mathbb{R}^{t1,n}_{incomplete} \end{cases} \tag{16}$$

Considering 16, we deduce $\exists[\alpha \in \mathbb{R}^{t1,n}_{complete} \wedge \alpha \notin \mathbb{R}^{t1,n}_{incomplete}]$. Applying theorem 1.2, the Heisenberg's Uncertainty principle is deduced to be a case of dimensional randomness. Such a limitation renders all potential physical laws that simultaneously use the position and momentum variables of a particle as invisible and inaccessible, making it necessary to use probability distributions to describe them.

5.3.4 Constraints on the hidden variables hypothesized in Bell's Theorem

Bell assumes that the orientation of the spins $\vec{\sigma}_1$ and $\vec{\sigma}_2$ of two particles is predetermined by a λ parameter [7]. λ is the representation of hidden variables hypothesized by Einstein in the foundations of Quantum Mechanics. In Bell's work, it is indifferent whether λ is a variable or a set of discrete or continuous variables, however there are no further explanations about what λ would be. Regarding the original work, we point out a new restriction on the distribution of the variables that make up the parameter λ. If the set of variables that compose λ belongs entirely to the accessible dimensions of spacetime, assuming realism, we know that under the observer's reference, an initial state determined by $\vec{\sigma}_1$, $\vec{\sigma}_2$ and λ must have only one possible result. However, if part of the variables of λ belong to the invisible and inaccessible dimensions of spacetime, applying theorem 1.1, we notice that the set of variables $\vec{\sigma}_1$, $\vec{\sigma}_2$ and λ may have more than one possible outcome for the observer referential, even assuming system realism. We will analyze the implication of differentiating the spacetime distribution of λ on the results of Bell's theorem. For such an analysis, we verified the implications of differentiating between invisible and inaccessible space on Schneider's combination test[33]. The test consists of defining, through the combination of all possible h, a lower limit for the probability that two observers Alice and Bob verify identical results for the polarization of two entangled photons. Due to the production in pairs of entangled particles, it is guaranteed that the pair of photons will have the same polarization if Alice and Bob take identically oriented polarizers. However, the test verifies what the expected behavior of the experiment should be by taking all 8 possible combinations of hidden variables assuming that Alice and Bob independently and randomly choose polarizers oriented between 3 different angles of 0º, 120º or 240º.

Taking Bell's notation, we assume that the λ parameter predetermines the positive or negative polarization of a particle. Although we don't know exactly what these variables are, we can know how much information they can carry as shown in the table 8.

In the table 8, the case column shows all 8 combinations that the λ parameter can take. For example, If λ corresponds to case 2, then we know that the spin of the particles will be positive for polarizers A and B, but negative for polarizer C. Column [AB] shows the case where Alice chose the polarizer A and Bob the polarizer B. Columns [BC] and [AC] vary only with respect to the polarizers chosen by Alice and Bob. Assuming Alice chose polarizer A and Bob polarizer B, we expect both to check for positive orientation, so column [AB] assumes a result of 1 for Case 2. However, if Alice chooses B and Bob C, Alice must observe the positive result and Bob the negative result. Because the results are different, the [BC] column assumes the value 0 in the

Table 8 Permutations of A, B and C with likelihood of matches (++ or –). Source: Schneider (2014)

Case	A=0º	B=120º	C=240º	[AB]	[BC]	[AC]	Average
1	A+	B+	C+	1 (++)	1 (++)	1 (++)	1.000
2	A+	B+	C-	1 (++)	0	0	.333
3	A+	B-	C+	0	0	1 (++)	.333
4	A+	B-	C-	0	1 (–)	0	.333
5	A-	B+	C+	0	1 (++)	0	.333
6	A-	B+	C-	0	0	1 (–)	.333
7	A-	B-	C+	1 (–)	0	0	.333
8	A-	B-	C-	1 (–)	1 (–)	1 (–)	1.000

line representing case 2. Finally, the average column presents the proportion of events in which Alice and Bob observe identical results for all possible combinations of hidden variables λ. Note that assuming a set of hidden variables λ, we expect a lower bound of $1/3$ for the equivalence of observables. However, the prediction of Quantum Mechanics for the occurrence of such equivalences is only 0.25, a result that is numerically incompatible with the existence of λ.

Notice that in the table there was an implicit assumption about what would be the hidden variables that make up λ. By assuming that a case has a single possible corresponding result, the experiment is assuming that all the variables that compose λ belong to the accessible space. However, if the variables of λ belong to the invisible and inaccessible space, the same initial conditions can have different results. Therefore, for any of the cases, the results can be positive or negative, not presenting a single determined state as shown in 8. A fact that imposes restrictions on the reasoning that demonstrates $1/3$ as the lower limit for observing the equivalence of results. This result is only valid disregarding the existence of invisible and inaccessible dimensions, however assuming these dimensions, the idea of local hidden variables λ is no longer incompatible with the results presented by Quantum Mechanics.

5.3.5 String Theory

String theory says that the fundamental constituent of matter are one-dimensional objects that would replace the point-particle model on which the standard model of particles and the quantum field theory are currently built [34]. But, a stable and consistent version of String Theory requires extra spatial dimensions in spacetime. M-Theory [35] describes the spacetime as eleven dimensional $\mathbb{R}^{1,10}_{complete}$. The explanation for why we do not experience the 7 additional dimensions is that those dimensions are compactified. Formally, the observable spacetime is $\mathbb{R}^{1,3}_{incomplete}$, but, in String Theory, the real spacetime is $\mathbb{R}^{1,10}_{complete}$. The universe description provided by String Theory is compliant with the expression $[\mathbb{R}^{t1,n}_{incomplete} \wedge \mathbb{R}^{t2,m}_{complete} \mid t1 + n < t2 + m]$. Applying theorem 1.1, if string theory is right, we expect that dimensional randomness would occur in a $\mathbb{R}^{1,3}_{incomplete}$ spacetime. The existence of extra dimensions makes String Theory a candidate hidden variable theory to explain the probabilistic nature of quantum mechanics.

6 Conclusions

Through this work, we distinguish between objective and subjective randomness based on the type of limitation encountered. Subjective randomness is a consequence of limitations inherent to the incompetence of an observer. On the other hand, objective randomness is a consequence of limitations imposed by the universe for the prediction of certain events, no matter how competent an observer may be. Dimensional randomness is subset of objective randomness, because it is not possible, under the observer's referential, to distinguish the class of events that have a fundamentally unknown cause from the class of events that are fundamentally absent of cause. The Thought Experiments were of paramount importance for these definitions, because they allowed us to use the closed world assumption and discriminate subjective limitations of objective limitations imposed by the systems s1 ,s2 ,s3 and s4. According to the definitions of the theory, it was possible to describe objective randomness for events beyond quantum mechanics, finding applications in astronomy, computing, and complex systems. The Theory of Dimensional Randomness is also important for computing, as understanding the mechanisms for generating objective randomness is an extremely relevant topic for information security, such as in the field of quantum cryptography. Understanding objective randomness is important beyond orthodox, empirical science. Applications of the Theory of Dimensional Randomness can also be extended to branches of metaphysics, such as in the quest to understand the existence of omniscience, free will and fate. Next steps for the theory aim to generalize the concept and application of invisible and inaccessible dimensions beyond the spatial and temporal definition, finding irreducible limitations beyond physics. Also the generalization of the Theory of Dimensional Randomness to a General Theory of randomness that explains completely the set of true random events. Finally, we believe that the theory already delimits the boundaries of empirical science in that it is able to differentiate between the fine line of accessible and inaccessible knowledge for the description of randomness.

7 Conflicts of interest

The author declares that there is not conflict of interest for the publication of this article.

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