Social Distancing and Coordination

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Abstract

The social distance critical to alleviate the spread of COVID-19 is quite insufficient. In order to contribute to the literature, "(1) When a rare virus such as COVID-19 appears, when the total level of social distance has natural inertia; (2) Open national public statements are essential to reduce this inertia and adjust the public's behavior to a new, optimal social distance level. (3) When individuals are not adequately informed about optimal social distance levels over time, national communication is more effective than local communication.
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INTRODUCTION

We focused on a model of social removal, motivated by the COVID-19 outbreak, where people value both joining the right social distance and complying with other people's social norms. When citizens consider these two issues and are unsure of each other's beliefs about the right social distance, collective social distancing shows significant inertia. In particular, by following the positive shock towards the correct social distance, the total social distances will be well below the social optimum, even if the information of the individuals reflects this shock correctly. (Proposition 3).
Citizens care to comply with social norms. For example, many make a strange shopping in a mask, risking disturbing a neighbor by waiting for the next elevator, rejecting an outstretched hand, or banning gaming games if other citizens are not doing the same. These concerns create inertia. actions of citizens ensure that their common knowledge of past social distance behaviors follows the social norms by coordinating the social distance. This inertia is problematic because society will be better off coordinating a new, higher social distance.

How can the state improve this situation? Our recommendation 4 shows that public statements about the new optimal level of social departure from leading leaders reduce inertia by generating new common information that allows people to coordinate their actions around new, more appropriate norms.

Should such statements be made public or more specifically? Proposition 5 shows that when optimal social distance levels have a high correlation over time and individuals are poorly informed, national communication is better than local communication, so that previous social distance norms are overweight.

This result reflects that the information transmitted publicly creates new common information.

Allcott et al. (2020) examines the perception of risk in the United States and the interaction between such externalities. The importance of citizens who sacrificed the interests of Dube and Baicker (2020) for greater well-being and Christensen et al. (2020) Examining the Ebola crisis in Sierra Leon highlights the importance of trust in local leaders and institutions.

In contrast, our analysis highlights the role of social norms and strategic uncertainty as a source of counter-productive inertia in collective social distance.

Government results are also very different. While information alone cannot solve the problem of standard externalities, clear and consistent public information can significantly increase social distance by reducing strategic uncertainty and enabling citizens to coordinate on new norms.

National networks produce more widespread information, enabling citizens to better coordinate on the new social withdrawal norms.

What sorts of decisions might our norms-based model represent in the context of the COVID-19 crisis? The key features of the model are that: (1) people are uncertain about the right level of social distancing and are uncertain about what others believe, (2) the right level of social distancing exhibits serial correlation, and (3) there is a desire for conformity with social norms.

An important application of our model concerns social gatherings. The advent of COVID-19 made it undesirable, from a social welfare perspective, for people to partici- pate in events such as St. Patrick's Day celebrations, spring break trips and parties, or sporting events. However, people were uncertain about the threat that COVID-19 posed. Crucially, to the extent that they believed others might think that the threat was modest and therefore continue to gather, they too had incentives to behave this way. The result was undesirable inertia—people continued past practices in ways that harmed social welfare.

Similar arguments hold for social practices such as hand shaking, kissing, and other forms of physical social greetings. It is awkward to refuse to shake an offered hand—as a recent presidential press briefing illustrated—creating a force for social conformity of the sort we model. The uncertainty about others' views on COVID-19 created unfortunate inertia in physical social greetings.

Our model also applies to some professional settings. If employees believe that continu- ing to come to the office signals commitment or ambition, then this creates social pressure for employees to continue coming to work if their managers or supervisors are. In the face of such pressure, the model suggests, there will be inertia that keeps people coming into the office at inefficiently high levels, especially during the early days of a viral outbreak[1]

Finally, the model also helps understand inertia in social distancing and its policy implications, not only at the initial outset of a contagious virus, but at any point where

[1]In this setting, there could also be congestion externalities that we leave unmodeled—as fewer people start going into the office, the risk of being infected in the office goes down

there is a sharp change in the appropriate level of social distancing.

We utilize a standard “beauty contest” model in which individuals aim to take actions that match an uncertain state of the world (the right action) while remaining close to the average action in the population. Subsequent literature explored the welfare consequences of this insight and its implications for optimal communication (Morris and Shin 2007; Angeletos and Pavan 2007). This framework has been applied to study a wide range of topics from monetary policy (Lorenzoni 2010) to leadership in party conferences (Dewan and Myatt 2007), organizations (Bolton et al. 2013), and judiciaries (Shadmehr et al. 2019). Our paper provides an application in disease control and public health policy.

Our paper is also related to the game theoretic literature on social norms. We adopt Acemoglu and Jackson’s (2017) definition of social norms as “the distribution of anticipated payoff-relevant behavior” (p. 246). Our model has a unique equilibrium, and social norms in our setting refer to the average (social distancing) behavior of the population. When agents are heterogeneously informed, the anticipated average behavior depends on the individual’s information. We model people’s desire to conform to social norms as a desire to do what is expected of them, as reflected in other people’s behavior. The challenge of a global pandemic is that it creates uncertainty about what people expect of one another and how people will behave, leading people to lean on past, inappropriate, norms of behavior.

In contrast to these papers, agents in our setting are uncertain about the right action and about one anothers’ information in a changing world. While Acemoglu and Jackson (2015, 2017) focus on patterns of cooperation and compliance, our focus is on inertia and over- reactions to public information in a world where optimal aggregate behavior evolves over time and on the resulting policy implications.

Model

We apply the canonical framework and results discussed in Angeletos and Lian's (2016) comprehensive review of the literature.[2] There is a continuum of citizens indexed by $i \in$

$[0, 1]$, interacting over time, indexed by $t = 0, 1, \ldots$. In each period t, each citizen must take an action $a_{it} \in \mathbb{R}$, which captures the degree of social distancing. A higher action corresponds to a higher level of social distancing. Absent concerns for conformity, the right action for each citizen is θ_t. But, in each period, each citizen cares about targeting this right action *and* about conforming to the average action that others take in that period,

$A_t = \int a_{it}\, di$. This generates a complementarity: if a citizen believes that others will do little social distancing, this raises that citizen's incentive also to do less social distancing. This captures, among other things, social pressure and the cost of deviating from the norms of behavior in the society. In particular, a citizen's payoff in period t is:

$$-(1-\alpha)(a_{it}-\theta_t)^2 - \alpha(a_{it}-A_t)^2, \tag{1}$$

where $\alpha \in (0,1)$ is the citizens' relative weight on conformity.
The right action, θ_t, follows a random walk: $\theta_t = \theta_{t-1} + u_t$, where $u_t \sim iidN(0, \sigma_u)$. Citizens do not observe θ_t, but each citizen observes a signal of the right action: $x_{it} = \theta_t + E_{it}$, where $E_{it} \sim iidN(0, \sigma_E)$. Throughout, we assume that the noise and fundamentals are independent from each other in the standard manner.

Citizen i observes x_{it} in period t, and θ_{t-1} becomes public in period t.[3] Citizens discount future payoffs by δ, and each citizen maximizes the expected sum of discounted period payoffs.

1 Analysis

We will think of the normative goal as each individual choosing the right level of aggregate social distancing, $a_{it} = \theta_t$, so that the aggregate level of social distancing is also right,

$A_t = \int a_{it} di = \theta_t$. This would be the objective of a policy maker who aggregates individual payoffs, but puts no weight on social conformity (i.e., sets $\alpha = 0$).[4] In light of this, we say

that any reduction in the expected quadratic distance between individual actions and the right

action, $E[\int (a_{it} - \theta_t)^2 di]$, is a *normative improvement.*

Our first result concerns the optimal level of social distancing under this normative crite- rion. Because the right action is uncertain, the normatively optimal individual actions—i.e.,

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[2]This literature builds on the seminal work of Morris and Shin (2002).

[3]Even if θ_{t-1} is not observed in the current period, citizens will infer it in equilibrium if they observe

the last period's aggregate behavior A_{t-1}.

[4]For our normative standard to coincide with the utilitarian social welfare, one can add a term $\alpha \int (a_{jt} - A_t)^2 dj$ to citizen payoffs in equation (1), so that citizens want the divergence of their own behavior from the average behavior to be close to the average divergence in the population.

the actions that minimize $E[\int (a_{it} - \theta_t)^2 di]$—involve citizens choosing their best estimate of the right action, $a_{it} = E[\theta_t | x_{it}, \theta_{t-1}] = \beta x_{it} + (1 - \beta)\theta_{t-1}$, where $\beta = \frac{\sigma_u^2}{\sigma_u^2 + \sigma^2}$.

Proposition 1 ***(Normative Benchmark)*** *If citizens did not care about conformity ($\alpha = 0$), the aggregate action would be: $A_t = \theta_t + \beta u_t$, where $\beta = \frac{\sigma_u^2}{\sigma_u^2 + \sigma^2}$.*

We now analyze how citizens actually behave, given their concern for taking the right action and for social conformity. Because there is a continuum of citizens, a citizen's action does not affect the aggregate outcome, either in the current or in future periods. Thus, the only link between periods is information. From equation (1), citizen i chooses the following degree of social distancing:

$$a_{it} = (1 - \alpha)\mathrm{E}_{it}[\theta_t] + \alpha\, \mathrm{E}_{it}[A_t], \tag{2}$$

where $\mathrm{E}_{it}[\cdot]$ is the expectation of i in period t given his information.

Define $\bar{\mathrm{E}}^h$ recursively as follows. $\bar{\mathrm{E}}^0[X] = X$, $\bar{\mathrm{E}}^1[X] = \bar{\mathrm{E}}[\bar{\mathrm{E}}^0[X]] = \int \mathrm{E}_i[X]di$, $\bar{\mathrm{E}}^h[X] = \bar{\mathrm{E}}[\bar{\mathrm{E}}^{h-1}[X]] = \int \mathrm{E}_i[\bar{\mathrm{E}}^{h-1}[X]]di$. That is, $\bar{\mathrm{E}}^1[X]$ is the average expectation of the random variable X in the population; $\bar{\mathrm{E}}^2[X]$ is the average expectation in the population about the average expectation in the population, and so on. Proposition 2 shows that the aggregate social distancing in the population depends on all such higher order expectations in the population. The proof comes from iterating on equation (2). (All proofs are in the appendix.)

Proposition 2 ***(Higher Order Beliefs in Aggregate Action)*** *The aggregate action in each period depends on all average higher order beliefs in the population about the right action, with lower weights on higher orders:*

$$A_t = \sum_{h=1}^{\infty} (1 - \alpha)\, \alpha^{h-1}\, \bar{\mathrm{E}}_t^h[\theta_t] \tag{3}$$

Observe that $\mathrm{E}_{it}[\theta_t] = \mathrm{E}_{it}[u_t] + \theta_{t-1} \Rightarrow$

$$\overline{\mathrm{E}}_t\,[\theta_t] = \overline{\mathrm{E}}_t\,[u_t] + \theta_{t-1}. \tag{4}$$

h h

Now, using properties of Normal distribution and Proposition 2 yields:

Proposition 3 ***(Aggregate Actions Exhibit Excess Inertia)*** *Conformity generates inertia. In particular,*

$\underline{(1-\alpha)\beta}$ σ^2

- $A_t = \theta_{t-1} + \varphi u_t$, *where* $0 < \varphi < \beta < 1$, $\varphi = {}_{1-\alpha\beta}$, *and* $\beta = {}_{\sigma}2\ 2$. $\frac{u}{}$

$u{+}\sigma$

- φ *is decreasing in* α, *with* $\lim_{\alpha\to 0} \varphi(\alpha) = \beta$ *and* $\lim_{\alpha\to 1} \varphi(\alpha) = 0$.

Citizens have common knowledge that, on average, the right action today is the right action yesterday (i.e., $\theta_t \sim N(\theta_{t-1}, \sigma_u)$) and hence over-weight this fact. As a result, today's aggregate action is biased in the direction of yesterday's right action. As a consequence, following a large positive shock, like COVID-19, to the right amount of social distancing, the aggregate action will be lower than the right action.[5]

Now, suppose in each period t, in addition to the their private signals x_{it}, citizens also receive a public signal $p_t = \theta_t + \eta_t$, with $\eta_t \sim iidN(0, \sigma_\eta)$. Such a public signal might be the result of information conveyed by the government. Proposition 2 and equation (4) still hold because they do not depend on the details of available information. However, the presence of public signals changes the degree of inertia in aggregate social distancing.

Proposition 4 ***(Public Announcements)***

1. *Public signals reduce inertia. Averaging over the public signal noise, the expected aggregate social distancing is:* $E[A_t|\theta_{t-1}, u_t] = \theta_{t-1} + \varphi_p u_t$, *where* $\varphi_p > \varphi$.

2. *The amount of inertia is decreasing in the clarity of the public signal. That is,* φ_p *is monotone decreasing in* σ_η, $\lim_{\sigma_\eta\to\infty} \varphi_p(\sigma_\eta) = \varphi$, *and* $\lim_{\sigma_\eta\to 0} \varphi_p(\sigma_\eta) = 1$.

3. *Improving the clarity of the public signal causes a normative improvement (i.e.,* $E[\int (a_{it} - \theta_t)^2 di]$ *is increasing in* σ_η*) if: (i)* $\alpha \le 1/2$ *or (ii)* σ_η *is sufficiently small.*

Proposition 4 shows that, following a shock, an informed leader can send a public signal that helps set public expectations about the aggregate right action, thereby reducing inertia in social distancing driven by the desire to conform. The clearer that message (i.e., the lower σ_η), the more this will reduce inertia.

Such public messages are a normative improvement if people don't put too much weight on conformity ($\alpha \le 1/2$) or the public signal is sufficiently informative (σ_η small). Why these conditions? Because citizens value conformity, they put excessive weight on all pub- lic signals relative to a Bayesian individual who only cares about choosing an action that reflects the best estimate of θ_t (this was the same logic that drove inertia in the first place). Because this distortion is smaller when α is smaller, new public information about the optimal social distancing is always beneficial when citizens put relatively less weight on conformity ($\alpha \le 1/2$). In the other extreme, when citizens almost only care about confor- mity ($\alpha \approx 1$), they put almost no weight on their private signals. Now, although citizens over-weight new public information (p_t), this over-reaction to the new public information helps counter-act their over-reaction to past experience (that $\theta_t \sim (\theta_{t-1}, \sigma_u)$), and the over- all effect is again beneficial. In between, when $\alpha \in (1/2, 1)$, these effects compete and the

[5] The over-weighting of public information is a key insight of Morris and Shin (2002) who showed that public announcements can be damaging to welfare, particularly, in financial settings.

overall effect of raising the precision of new public information may be negative unless it is sufficiently informative (σ_η small) to offset the over-reaction.[6]

For social distancing in the presence of a dangerous infectious disease, we believe the relevant parameter space is $\alpha \leq 1/2$. It is unlikely that people care so much about conformity that over-reaction to new public information trumps its value. Hence, for cases like COVID-19, Proposition 4 suggests that clear and consistent public messages from a leader are socially beneficial.

Given the overreaction by citizens to public messages described above, one may wonder whether there is a better way for the government to deliver information. Would it be better for citizens to receive the same level of information, but privately rather than publicly? For instance, perhaps employers or local governments could provide private information to citizens, rather than them all observing the same public information in a presidential speech or press conference.

To consider this possibility, contrast the public signal case with a setting where, instead of receiving private and public signals $x_{it} \sim N(\theta_t, \sigma^2_E)$ and $p_t \sim N(\theta_t, \sigma^2_\eta)$, citizens receive a single private signal x^i_{it} with the same amount of information about the right action θ_t as the public and private signals combined. In particular, let $x^i_{it} = \theta_t + E^i_{it}$, with $E^i_{it} \sim N(0, \sigma^2_E = \frac{\sigma^2_E \sigma^2_\eta}{\sigma^2_E + \sigma^2_\eta})$.

Proposition 5 *The setting with the combination of private and public signals* (x_{it}, p_t) *is a normative improvement over the setting with more precise private signals* x^i_{it} *when* σ_u *is* *sufficiently small or σ_E is sufficiently large.*

When citizens believe the past is highly informative about the present (σ_u small) or that they are privately poorly-informed (σ_E large), citizens put too much weight on their past experience. In such circumstances, it is better for the government to communicate publicly rather than privately. Citizens over-react to the government's public messages. But that will help to counter-act their over-reaction to their past experience. By contrast, when citizens believe the past is relatively uninformative (σ_u large) or that they are privately well-informed (σ_E small), the government should communicate privately.

In the context of social distancing in the wake of a new and rare infectious disease, individuals' information is typically very noisy (σ_E is large) and the appropriate level of social distancing is very sticky (σ_u is low, the disease is a very unusual shock). As such, for social distancing in the wake of COVID-19, clear and consistent public statements by a national leader are more effective than statements by local governments or employers, not because the national government is more informed, but because clear national statements generate over-reaction that is beneficial to correct the inertia created by the over-weighting of past habits and social norms.

[6] Equation (16) in the proof of Proposition 4 shows the necessary and sufficient conditions for when reducing σ_η is a normative improvement.

3 References

Acemoglu, Daron, and Matthew O. Jackson. 2015. "History, Expectations, and Leadership in the Evolution of Social Norms." *Review of Economic Studies* 82: 1-34.

Acemoglu, Daron, and Matthew O. Jackson. 2017. "Social Norms and the Enforcement of Laws." *Journal of the European Economic Association* 12: 245-95.

Acemoglu, Daron, and Alexander Wolitzky. 2014. "Cycles of Conflict: An Economic Model." *American Economic Review* 104: 1350-67.

Allcott, Hunt, Levi Boxell, Jacob Conway, Matthew Gentzkow, Michael Thaler, and David Yang. 2020. "Polarization and Public Health: Partisan Differences in Social Distancing during the Coronavirus Pandemic." Mimeo.

Angeletos, George-Marios, and Alessandro Pavan. 2007. "Efficient Use of Information and Social Value of Information." *Econometrica* 75: 1103-42.

Angeletos, George-Marios, and Chen Lian. 2016. "Incomplete Information in Macroeco- nomics: Accommodating Frictions in Coordination." *Handbook of Macroeconomics, Vol. 2*: 1065-1240.

Bolton, Patrick, Markus K. Brunnermeier, and Laura Veldkamp. 2013. "Leadership, Co- ordination and Corporate Culture." *Review of Economic Studies* 80: 512-37.

Christensen, Darin, Oeindrila Dube, Johannes Haushofer, Bilal Siddiqi, and Maarten Voors. 2020.

"'Building Resilient Health Systems: Experimental Evidence from Sierra Leone and the 2014 Ebola Outbreak." Mimeo.

Dewan, Torun, and David P. Myatt. 2007. "Leading the Party: Coordination, Direction, and Communication." *American Political Science Review* 101: 827-45.

Dube, Oeindrila, and Katherine Baicker. 2020. "How You Can Protect Your Community, Not Just Your Own Health." *New York Times, March 26, 2020.*

Lorenzoni, Guido. 2010. "Optimal Monetary Policy with Uncertain Fundamentals and Dispersed Information. *Review of Economic Studies* 77: 305-38.

Morris, Stephen, and Hyun Song Shin. 2002. "The Social Value of Public Information." *American Economic Review* 92: 1521-34.

Morris, Stephen, and Hyun Song Shin. 2007. "Optimal Communication." *Journal of the European Economic Association* 5: 594-602.

Shadmehr, Mehdi, Charles Cameron, and Sepehr Shahshahani. 2019. "Coordination and Innovation in Judiciaries: Correct Law vs. Consistent Law." Mimeo.

4 Appendix: Proofs

Proof of Proposition 2: From equation (2),

$$A_t = \int a_{it} di = \int ((1-\alpha)\mathrm{E}_{it}[\theta_t] + \alpha\, \mathrm{E}_{it}[A_t]) di = (1-\alpha)\bar{\mathrm{E}}_t[\theta_t] + \alpha\, \bar{\mathrm{E}}_t[A_t].$$

Iterating yields:

$$A_t = (1-\alpha)\bar{\mathrm{E}}_t[\theta_t] + (1-\alpha)\alpha\, \bar{\mathrm{E}}_t^2[\theta_t] + \alpha^2 \bar{\mathrm{E}}_t^2[A_t].$$

Repeated iteration yields the result. $\square$

Proof of Proposition 3: We calculate $\bar{\mathrm{E}}_t^h[\theta_t]$, and use Proposition 2. Note that $x_{it} = \theta_t + E_{it} = \theta_{t-1} + u_t + E_{it}$. Thus, letting $\beta = \frac{\sigma_u^2}{\sigma_u^2 + \sigma^2}$,

$$\mathrm{E}_{it}[u_t] = \beta(x_{it} - \theta_{t-1}) = \beta(u_t + E_{it}) \Rightarrow \bar{\mathrm{E}}_t[u_t] = \beta u_t.$$

Iterating yields:

$$\bar{\mathrm{E}}_t^h[u_t] = \beta^h u_t. \tag{5}$$

Substituting from equation (5) into equation (4) yields:

$$\bar{\mathrm{E}}_t^h[\theta_t] = \beta^h u_t + \theta_{t-1}. \tag{6}$$

Now, substituting from equation (6) into equation (3) in Proposition 2 yields:

$$A_t = \sum_{h=1}^{\infty} (1-\alpha)\, \alpha^{h-1} \, (\beta^h \, u_t + \theta_t$$

$-1) = \theta$

$t-1$ +

$$\frac{\beta(1-\alpha)}{1-\alpha\beta} u_t. \qquad (7)$$

In equation (7), let $\varphi = \frac{(1-\alpha)\beta}{1-\alpha\beta}$, and observe that $\lim_{\sigma_E \to 0} \beta = \lim_{\sigma_u \to \infty} \beta = 1$. □

Proof of Proposition 4: *Part 1.* With the public signal p_t, $\mathrm{E}_{it}[u_t] = E[u_t | x_{it}, p_t]$. Thus,

$$\mathrm{E}_{it}[u_t] = \frac{\sigma_u^2\sigma_\eta^2 (x_{it} - \theta_{t-1}) + \sigma_u^2\sigma_E^2 (p_t - \theta_{t-1})}{\sigma_u^2\sigma_\eta^2 + \sigma_u^2\sigma_E^2 + \sigma_E^2\sigma_\eta^2}$$

$$= \frac{\sigma_u^2\sigma_\eta^2 (u_t + E_{it}) + \sigma_u^2\sigma_E^2 (p_t - \theta_{t-1})}{\sigma_u^2\sigma_\eta^2 + \sigma_u^2\sigma_E^2 + \sigma_E^2\sigma_\eta^2}. \qquad (8)$$

Thus,

$$\bar{\mathrm{E}}_t[u_t] = \frac{\sigma_u^2\sigma_\eta^2 u_t + \sigma_u^2\sigma_E^2 (p_t - \theta_{t-1})}{\sigma_u^2\sigma_\eta^2 + \sigma_u^2\sigma_E^2 + \sigma_E^2\sigma_\eta^2} = A_u\, u_t + A_p\, (p_t - \theta_{t-1}), \qquad (9)$$

where

$$A_u = \frac{\sigma_u^2\sigma_\eta^2}{\sigma_u^2\sigma_\eta^2 + \sigma_u^2\sigma_E^2 + \sigma_E^2\sigma_\eta^2} \text{ and } A_p = \frac{\sigma_u^2\sigma_E^2}{\sigma_u^2\sigma_\eta^2 + \sigma_u^2\sigma_E^2 + \sigma_E^2\sigma_\eta^2}, \qquad (10)$$

with

$$\lim_{\sigma_\eta \to \infty} A_p = 0, \quad \lim_{\sigma_\eta \to \infty} A_u = \beta, \quad \lim_{\sigma_\eta \to 0} A_p = 1, \text{ and } \lim_{\sigma_\eta \to 0} A_u = 0. \qquad (11)$$

Iterating on equation (9) yields

$$\bar{E}_t^h[u_t] = (A_u)^h u_t + (1 + \cdots + A_u^{h-1})A_p(p_t - \theta_{t-1}). \tag{12}$$

Substituting from equation (12) into equation (4) yields:

$$\begin{aligned}
\bar{E}_t^h[\theta_t] &= (A_u)^h u_t + (1 + \cdots + A_u^{h-1})A_p(p_t - \theta_{t-1}) + \theta_{t-1} \\
&= (A_u)^h u_t + \frac{1 - A_u^h}{1 - A_u} A_p(p_t - \theta_{t-1}) + \theta_{t-1}.
\end{aligned} \tag{13}$$

Now, substituting from equation (13) into equation (3) in Proposition 2 yields:

$$\begin{aligned}
A_t &= \sum_{h=1}^{\infty} (1-\alpha)\,\alpha^{h-1} \left((A_u)^h u_t + \frac{1 - A_u^h}{1 - A_u} A_p(p_t - \theta_{t-1}) + \theta_{t-1} \right) \\
&= \theta_{t-1} + \frac{(1-\alpha)A_u}{1-\alpha A_u} u_t + (1-\alpha)\frac{A_p}{1-A_u}\left(\frac{1}{1-\alpha} - \frac{A_u}{1-\alpha A_u} \right)(p_t - \theta_{t-1}) \\
&= \theta_{t-1} + \frac{(1-\alpha)A_u}{1-\alpha A_u} u_t + \frac{A_p}{1-\alpha A_u}(p_t - \theta_{t-1}) \\
&= \theta_{t-1} + \frac{(1-\alpha)A_u u_t + A_p(p_t - \theta_{t-1})}{1-\alpha A_u}.
\end{aligned} \tag{14}$$

Note that, using (11), if $\sigma_\eta \to \infty$, equation (14) simplifies to equation (7).
For given θ_{t-1} and u_t, aggregate action A_t takes different values for different values of the public signal p_t, depending on the idiosyncratic error term η_t in the public signal. The

average public signal, for given θ_{t-1} and u_t, is $E[p_t|u_t] = \theta_{t-1} + u_t$. Then, averaging over the public signal noise, equation (14) becomes:

$$E[A_t|u_t,\theta_{t-1}] = \theta_{t-1} + \varphi_p u_t, \text{ where } \varphi_p = \frac{(1-\alpha)A_u + A_p}{1-\alpha A_u}.$$

Part 2. From (11), $\lim_{\sigma_\eta \to 0} \varphi_p = 1$ and $\lim_{\sigma_\eta \to \infty} \varphi_p = \varphi$. Comparing with φ in Proposition 3 yields:

$$\varphi_p - \varphi = \frac{(1-\alpha)A_u + A_p}{1-\alpha A_u} - \frac{(1-\alpha)\beta}{1-\alpha\beta} = \frac{\sigma_E^4\sigma_u^2}{(\sigma_E^2 + (1-\alpha)\sigma_u^2)(\sigma_E^2\sigma_u^2 + \sigma_\eta^2(\sigma_E^2 + (1-\alpha)\sigma_u^2))} > 0.$$

That is, $\varphi < \varphi_p$. Moreover,

$$\frac{d\varphi_p}{d\sigma_\eta^2} = -\frac{\sigma_E^4\sigma_u^2}{(\sigma_E^2\sigma_u^2 + \sigma_\eta^2(\sigma_E^2 + (1-\alpha)\sigma_u^2))^2} < 0.$$

Thus, reducing the noise in the public signal (less σ_η^2) raises φ_p.

Part 3. From (2),

$$\begin{aligned}
a_{it} &= (1-\alpha)E_{it}[\theta_t] + \alpha E_{it}[A_t] \\
&= (1-\alpha)E_{it}[\theta_{t-1} + u_t] + \alpha E_{it}\left(\theta_{t-1} + \frac{(1-\alpha)A_u u_t + A_p(p_t - \theta_{t-1})}{1-\alpha A_u}\right) && \text{(from (14))} \\
&= \theta_{t-1} + (1-\alpha)E_{it}[u_t] + \alpha\frac{(1-\alpha)A_u}{1-\alpha A_u}E_{it}[u_t] + \alpha\frac{A_p(p_t-\theta_{t-1})}{1-\alpha A_u} \\
&= \theta_{t-1} + \frac{1-\alpha}{1-\alpha A_u}E_{it}[u_t] + \alpha\frac{A_p}{1-\alpha A_u}(p_t - \theta_{t-1}) \\
&= \theta_{t-1} + \frac{1-\alpha}{1-\alpha A_u}(A_u(u_t + E_{it}) + A_p(p_t - \theta_{t-1})) + \alpha\frac{A_p}{1-\alpha A_u}(p_t - \theta_{t-1}) && \text{(from (8) and (10))}
\end{aligned}$$

$(1-\alpha)A_u$ $(1-\alpha)A$ A

$$= \theta_{t-1} + \quad 1-\alpha A_u \quad u_t + \quad \frac{u E_{it}}{1-\alpha A_u +} \quad \frac{p}{1-\alpha A_u} \; (p_t - \theta_{t-1}).$$

Thus,

$$a_{it} - \theta_t = \frac{(1-\alpha)A_u}{1-\alpha A_u} u_t + \frac{(1-\alpha)A_u}{1-\alpha A_u} E_{it} + \frac{A_p}{1-\alpha A_u}(p_t - \theta_{t-1}) - u_t$$

$$= \frac{(A_p + A_u - 1)u_t + A_p \eta_t + (1-\alpha)A_u E_{it}}{1-\alpha A_u} \quad \text{(substituting } p_t - \theta_{t-1} = u_t + \eta_t\text{)}.$$

Thus,

$$(A_p + A_u - 1)^2 u^2 + A^2 \eta^2 + (1-\alpha)^2 A^2 \qquad E^2$$

$$(a_{it} - \theta_t)^2 \quad = \quad \rule{6cm}{0.4pt}$$

$$+\frac{2(A_p+A_u-1)u_tA_p\eta_t+2(A_p+A_u-1)u_t(1-\alpha)A_uE_{it}+2A_p\eta_t(1-\alpha)A_uE_{it}}{(1-\alpha A_u)^2}.$$

Thus,

$$\int(a_{it}-\theta_t)^2di=\frac{(A_p+A_u-1)^2u_t^2+A_p^2\eta_t^2+(1-\alpha)^2A_u^2\sigma_E^2+2(A_p+A_u-1)A_p u_t\eta_t}{(1-\alpha A_u)^2}.$$

Thus,

$$E\left[\int(a_{it}-\theta_t)^2di\right]=\frac{(A_p+A_u-1)^2\sigma_u^2+A_p^2\sigma_\eta^2+(1-\alpha)^2A_u^2\sigma_E^2}{(1-\alpha A_u)^2},\qquad(15)$$

where we recognize that if $\alpha=0$, equation (15) simplified to $\frac{\sigma_E^2\sigma_u^2\sigma_\eta^2}{\sigma_u^2\sigma_\eta^2+\sigma_u^2\sigma_E^2+\sigma_E^2\sigma_\eta^2}$, which is the variance of $\theta_t|\theta_{t-1},p_t,x_{it}$. Differentiating with respect to σ_η^2 yields:

$$\frac{dE\left[\int(a_{it}-\theta_t)^2di\right]}{d\sigma_\eta^2}=\frac{\sigma_E^4\sigma_u^4(\sigma_E^2\sigma_u^2+\sigma_E^2\sigma_\eta^2+(1-\alpha)(1-2\alpha)\sigma_u^2\sigma_\eta^2)}{(\sigma_E^2\sigma_u^2+\sigma_\eta^2\sigma_E^2+(1-\alpha)\sigma_\eta^2\sigma_u^2)^3}.\qquad(16)$$

Thus, if $\sigma_E^2+(1-\alpha)(1-2\alpha)\sigma_u^2\geq 0$ (in particular, if $\alpha\leq 1/2$), the above derivative is strictly positive. If, instead, $\sigma_E^2+(1-\alpha)(1-2\alpha)\sigma_u^2<0$, the above derivative is strictly positive if and only if σ_η^2 is sufficiently small. $\square$

Proof of Proposition 5: To obtain $E\left[\int(a_{it}-\theta_t)^2di\right]$ with only x_t^i, first let $\sigma_\eta\to\infty$ in (15), and then substitute σ_E^2 with $\frac{\sigma_E^2\sigma_\eta^2}{\sigma_E^2+\sigma_\eta^2}$. Using (11), and recognizing that $\lim_{\sigma_\eta\to\infty}A_p\sigma_\eta^2=0$, the first step yields:

$$\lim_{\sigma_\eta \to \infty} \frac{(\beta-1)^2\sigma_E^2 + (1-\alpha)^2\beta^2\sigma_u^2}{(1-\alpha\beta)^2} = \frac{\sigma_E^2\sigma_u^2(\sigma_E^2+(1-\alpha)^2\sigma_u^2)}{(\sigma_E^2+(1-\alpha)\sigma_u^2)^2}.$$

Substituting σ_E^2 with σ_u^2 yields:

$$E\,[\cdot] = \frac{\sigma_E^2\sigma_u^2(\sigma_E^2+(1-\alpha)^2\sigma_u^2)}{(\sigma_E^2+(1-\alpha)\sigma_u^2)^2}. \tag{17}$$

Now, subtracting (15) from (17) yields:

$$\Delta = E\left[\int (a_{it}-\theta_t)^2 di \,\Big|\, x_{it}\right] - E\left[\int (a_{it}-\theta_t)^2 di \,\Big|\, x_{it}, p_t\right]$$

$$= \frac{\alpha^2\sigma_\eta^4\sigma_E^4\sigma_u^4}{(\sigma_\eta^2\sigma_E^2+(1-\alpha)(\sigma_\eta^2+\sigma_E^2)\sigma_u^2)^2(\sigma_E^2\sigma_u^2+\sigma_\eta^2(\sigma_E^2+(1-\alpha)\sigma_u^2))^2}\left(\sigma_E^4(\sigma_\eta^2+\sigma_u^2)-(1-\alpha)^2(\sigma_\eta^2+\sigma_E^2)\sigma_u^4\right).$$

As expected, $\lim_{\alpha\to 0}\Delta = 0$, because only the amount information matter; and $\lim_{\alpha\to 1}\Delta > 0$, because then citizens put a lot of weight of the pre-existing public information from the previous period, which need to be countered by new public information about θ_t. Moreover, for any $\alpha > 0$, the setting with both public and private signals (x_{it}, p_t) is a normative improvement over the setting with only private signals (x_i^i) if and only if $\Delta > 0$, i.e., if and only if

$$\left(\frac{\sigma_E^2}{\sigma_u^2}\right)^2 > (1-\alpha)^2\frac{\sigma_\eta^2+\sigma_E^2}{\sigma_\eta^2+\sigma_u^2}.$$

The result follows from inspection of this inequality. D