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Numerical Methods for Uncertainty Estimation in Mechanical Systems: A Literature Review

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Abstract

Uncertainty and reliability analyses are essential for evaluating the performance and safety of engineering systems subject to variability in parameters, operating conditions, and modeling assumptions. This paper presents a comprehensive review of classical and modern approaches for uncertainty quantification and reliability assessment. Three main methodological frameworks are addressed: probabilistic, fuzzy, and interval-based methods. Probabilistic approaches, such as the Monte Carlo Simulation (MCS), First-Order Reliability Method (FORM), and Second-Order Reliability Method (SORM), are discussed as rigorous tools for modeling randomness when sufficient statistical data are available. Fuzzy and interval methods are examined as alternatives to represent epistemic uncertainty when probabilistic information is limited or unavailable. The comparative analysis highlights the advantages, limitations, and applicability of each method in engineering contexts. The results emphasize that the choice of approach depends on the nature of uncertainty, data availability, and computational constraints. The study concludes that integrating probabilistic and non-probabilistic frameworks offers a promising pathway toward more robust and interpretable uncertainty analysis in complex systems.

Keywords: Uncertainty Analysis, Reliability, Robust Design, Mechanical Systems

1 Introduction

Uncertainty quantification (UQ) refers to the systematic process of identifying, characterizing, and propagating uncertainties in mathematical and computational models of physical systems. In contrast to reliability analysis—which primarily evaluates the likelihood of failure—UQ aims to provide a comprehensive representation of how uncertainties in model parameters, boundary conditions, or input data affect system responses. The overarching goal is to improve confidence in model predictions and to support robust design and decision-making under uncertainty [1].

Uncertainties can be broadly categorized into two types:

- Aleatory uncertainty, also called stochastic or inherent variability, represents natural randomness associated with physical phenomena, such as material properties, environmental conditions, or operational loads.
- Epistemic uncertainty, associated with incomplete knowledge, modeling simplifications, or insufficient data, which can in principle be reduced with additional information.

In classical uncertainty quantification, random variables

$$\mathbf{X} = (X_1, X_2, \ldots, X_n) \tag{1}$$

are used to describe uncertain model parameters, each characterized by a probability density function (PDF), cumulative distribution function (CDF), and statistical moments. When probabilistic information is scarce, non-probabilistic frameworks—such as interval analysis, fuzzy set theory, or evidence theory—can be employed to represent uncertainty in a more general or less data-demanding form.

The process of UQ generally comprises four main stages [2]:

1. Identification: defining the main sources and types of uncertainty;
2. Representation: modeling uncertainties via probability distributions, intervals, or fuzzy variables;
3. Propagation: evaluating how uncertainties in inputs affect outputs through analytical, numerical, or surrogate-based methods;
4. Reduction: mitigating the effects of uncertainty through sensitivity analysis, calibration, or robust optimization.

Through these stages, UQ provides a quantitative understanding of model credibility, enabling engineers and scientists to assess the variability and confidence associated with simulation results, thus supporting more reliable predictions and informed design decisions.

In this manuscript, different approaches for uncertainty and reliability analysis are investigated, emphasizing probabilistic, fuzzy, and interval-based methods. The theoretical foundations of each approach are presented and compared, highlighting their assumptions, advantages, and limitations in engineering applications. Particular attention is given to classical probabilistic techniques such as the Monte Carlo Simulation (MCS), First-Order Reliability Method (FORM), and Second-Order Reliability Method (SORM), as well as to non-probabilistic frameworks that address epistemic uncertainty through fuzzy and interval representations.

The remainder of this paper is organized as follows. Section 2 presents the main concepts and formulations of uncertainty analysis, including probabilistic, fuzzy, and interval methods. Section 3 discusses reliability assessment techniques under these different uncertainty representations. Section 4 shows the main definitions of robust design. Section 5 presents several applications of the numerical methods to mechanical structures and mechanisms. Finally, Section 7 summarizes the key findings and provides concluding remarks on the applicability and future perspectives of uncertainty and reliability analysis methods.

2 Uncertainty Analysis

Uncertainty analysis aims to systematically assess how imprecision, variability, and incomplete knowledge in model parameters or inputs affect the predicted outputs of engineering and scientific systems. Several methodological frameworks have been developed to represent and propagate uncertainties, each with its own assumptions and applicability. Probabilistic methods rely on probability theory to characterize randomness, fuzzy methods are based on fuzzy set theory to capture vagueness and imprecision, and interval methods employ interval arithmetic to handle bounded but non-probabilistic uncertainty. Together, these approaches provide complementary tools for understanding and mitigating the impact of uncertainty in modeling and decision-making.

2.1 Probabilistic Methods

Classical methods for uncertainty analysis are grounded in probability theory, where uncertainties are modeled as random variables or random processes with known probability distributions. The goal is to characterize how input uncertainties propagate through a system model and affect the output performance or reliability. Three main classes of probabilistic techniques are highlighted: probability theory, Monte Carlo simulation, and local expansion-based methods.

2.1.1 Probability Theory

In probabilistic modeling, uncertainties are described by probability density functions (PDFs), cumulative distribution functions (CDFs), and their associated statistical moments (mean, variance, skewness, etc.). Given a performance function $g(\mathbf{X})$ depending on a set of random variables $\mathbf{X} = (X_1, X_2, \ldots, X_n)$, the distribution of the output can be derived through analytical transformations, whenever feasible. This framework provides the basis for reliability analysis, where the probability of failure is often expressed as

$$P_f = \mathbb{P}\big[g(\mathbf{X}) \leq 0\big]. \tag{2}$$

2.1.2 Monte Carlo Simulation

Monte Carlo simulation (MCS) is a widely used numerical method for propagating uncertainties. The basic idea is to generate a large number N of random samples of the input vector $\mathbf{X}$ according to their joint probability distribution, evaluate the corresponding outputs $g(\mathbf{X}_i)$, and then approximate the statistics of interest. For instance, the failure probability can be estimated as

$$\hat{P}_f = \frac{1}{N}\sum_{i=1}^{N} I\big(g(\mathbf{X}_i) \leq 0\big), \tag{3}$$

where $I(\cdot)$ is the indicator function. Although conceptually simple and highly flexible, MCS is computationally expensive, especially for rare-event probability estimation or for complex models with high evaluation costs.

2.1.3 Local Expansion-Based Methods

To reduce computational burden, local approximation methods expand the performance function $g(\mathbf{X})$ around a reference point (often the mean value of the random variables). Two common techniques are:

Taylor Series Expansion.

The performance function can be approximated by a Taylor expansion around the mean $\boldsymbol{\mu}$:

$$g(\mathbf{X}) \approx g(\boldsymbol{\mu}) + \nabla g(\boldsymbol{\mu})^T(\mathbf{X} - \boldsymbol{\mu}) + \tfrac{1}{2}(\mathbf{X} - \boldsymbol{\mu})^T H(\boldsymbol{\mu})(\mathbf{X} - \boldsymbol{\mu}), \tag{4}$$

where ∇g is the gradient vector and H is the Hessian matrix of second derivatives. This allows estimation of output mean and variance using input statistics without resorting to full simulation.

Perturbation Method.

The perturbation method is a special case of Taylor expansion that truncates higher-order terms, typically retaining only the linear or quadratic terms. It is efficient for small input variances and smooth performance functions, providing analytical approximations of output moments. However, its accuracy deteriorates when the system exhibits strong nonlinearities or when uncertainties are large.

In summary, classical probabilistic approaches provide a rigorous framework for uncertainty quantification. Probability theory establishes the foundations, Monte Carlo simulation offers generality at the expense of high computational cost, and local expansion-based methods provide efficient but approximate solutions.

2.2 Interval Methods

The interval problem consider, related to interval analysis, consists of calculating the variation in the output of a model of a system represented by the function f applied to a set of parameters $\mathbf{p}$ [3]. The relationship between the input parameters and the output y is defined by:

$$y = f(\mathbf{p}) \tag{5}$$

The main objective of interval analysis is to calculate the variation of the output y of the system model considering that the set of parameters $\mathbf{p}$ can vary between a lower bound $\mathbf{p}_l$ and an upper bound $\mathbf{p}_r$. Consequently:

$$\bar{y} = \bar{f}(\bar{\mathbf{p}}) \tag{6}$$

with:

$$\bar{\mathbf{p}} = [\mathbf{p}_l, \mathbf{p}_r] \tag{7}$$

$$\bar{y} = [y_l, y_r] \tag{8}$$

$\bar{f}$ is the interval function that represents the relationship between the input interval and the output interval.

Two main strategies have been used to calculate the interval output $\bar{y}$: *i*) global optimization [4] and *ii*) interval arithmetic [5]. In this work, the first strategy will be used: global optimization.

In global optimization, the resulting interval is calculated by maximizing and minimizing the objective function f in the domain $\bar{\mathbf{p}}$ corresponding to the uncertain parameters, thus:

$$y_l = \min_{\mathbf{p} \in \bar{\mathbf{p}}} f(\mathbf{p}) \tag{9}$$

$$y_l = \max_{\mathbf{p} \in \bar{\mathbf{p}}} f(\mathbf{p}) \tag{10}$$

The strategy allows you to find the upper and lower bounds of the output variation. However, the computational cost can be very high, especially when analyzing systems with complex computational models and many uncertain parameters.

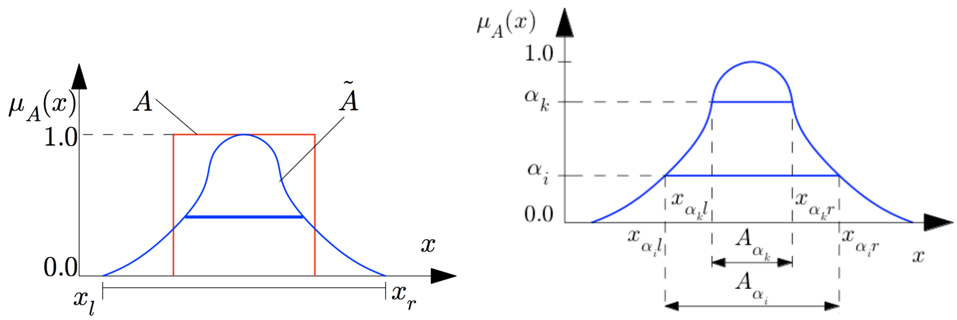
2.3 Fuzzy Methods

In several cases, some parameters of the systems can not be accurately estimated due to small variations around its nominal value. In these cases, these parameters can be modeled by means of fuzzy variables. As mentioned above, the fuzzy set theory was initially formulated by [6] to represent vague or ambiguous information. Thereby, it is possible to represent inaccurate or uncertain parameters by using fuzzy variables, specially when the stochastic process which models the uncertain parameter is unknown.

[7] represented a fuzzy variable as intervals weighted by a membership function using the α-level representation. In this contribution, the methodology to analyze the fuzzy uncertainties is based on α-level technique presented by [8]. The basic concepts of the fuzzy variables are introduced next.

2.3.1 Fuzzy Variables

Let $\mathbf{X}$ be an universal classical set of objects whose generic elements are denoted x. The subset A (where, $A \in \mathbf{X}$) is defined by the classical membership function $\mu_A : \mathbf{X} \rightarrow \{0, 1\}$ (see Fig. 1(a)). Furthermore, a fuzzy set $\tilde{A}$ is defined by means of the membership function $\mu_A : \mathbf{X} \rightarrow [0, 1]$, where $[0, 1]$ is a continuous interval. The membership function indicates the degree of compatibility of the element x to $\tilde{A}$. The closer the value of $\mu_A(x)$ to "1", the more x belong to $\tilde{A}$.



(a) Fuzzy set. (b) α-levels.

Fig. 1 Fuzzy set and α-level representation.

Thus, the fuzzy set is completely defined by:

$$\tilde{A} = \{(x, \mu_A(x)) \,|x \in \mathbf{X}\}, \text{where, } 0 \leq \mu_{\mathbf{A}} \leq 1 \tag{11}$$

For computational purposes, the fuzzy set $\tilde{A}$ can be represented by means of subsets thath are denominated α-levels. These subsets, which correspond to real and continuous intervals, are defined by A_{α_k} (see Fig. 1(b)), thus:

$$A_{\alpha_k} = \{x \in \mathbf{X}, \mu_{\mathbf{A}}(x) \geq \alpha_k\} \tag{12}$$

The α-level subsets of $\tilde{A}$ have the property:

$$\underline{A}_{\alpha_k} \subseteq \underline{A}_{\alpha_i} \forall \alpha_i, \alpha_k \in (0, 1] \quad \text{with} \quad \alpha_i \leq \alpha_k \tag{13}$$

If the fuzzy set is convex (in the unidimensional case), each α-level subset A_{α_k} corresponds to the interval $[x_{\alpha_k l}, x_{\alpha_k r}]$ where:

$$\begin{aligned} x_{\alpha_k l} &= \min[x \in \mathbf{X} | \mu_A(x) \geq \alpha_k] \\ x_{\alpha_k r} &= \max[x \in \mathbf{X} | \mu_A(x) \geq \alpha_k] \end{aligned} \tag{14}$$

2.3.2 Fuzzy Random Variables

The fuzzy random variables were defined perviously by [9]. According to the probability theory, the space of random elementary events Ω is presented. A fuzzy realization of the form $\underline{\tilde{x}}(\omega) = \tilde{x} \subseteq \underline{X}$ is assigned to each elementary event $\omega \in \Omega$. Accordingly, a fuzzy random variable $\underline{\tilde{X}}$ is the fuzzy result of the uncertain mapping

$$\Omega \rightarrow \mathbf{F}(\mathbb{R}^n) \tag{15}$$

where $\mathbf{F}(\mathbb{R}^n)$ is the set of all fuzzy numbers in $\mathbb{R}^n$. Each real random variable $\underline{X}$ on $\underline{\mathbf{X}}$ is contained in $\underline{\tilde{X}}$.

Based on this definition, a fuzzy random variable $\underline{\tilde{X}}$ can be described by a fuzzy cumulative distribution function $\tilde{F}(x)$. The fuzzy cumulative distribution function $\tilde{F}(\underline{x})$ of $\underline{\tilde{X}}$ is the set of probability distribution function of all originals $\underline{X}_j$ of $\underline{\tilde{X}}$ with membership values $\mu(F(\underline{\tilde{x}}))$. As a result, a fuzzy functional value $\tilde{F}(x_i)$ belongs to each value x_i (see Fig. 2(a)). The fuzzy probability distribution function $\tilde{F}(\underline{x})$ can also also be written in a parametric form (see Fig. 2(b)).

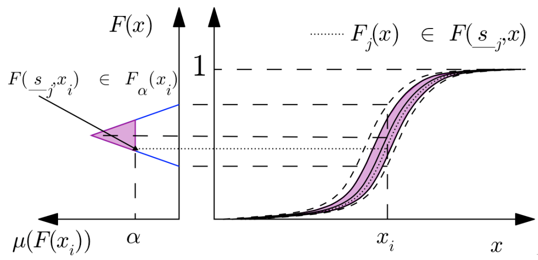
$$\tilde{F}(\underline{x}) = F(\tilde{s}, \underline{x}) \tag{16}$$

Thereby, for numerical computation, the α-discretization is properly used.

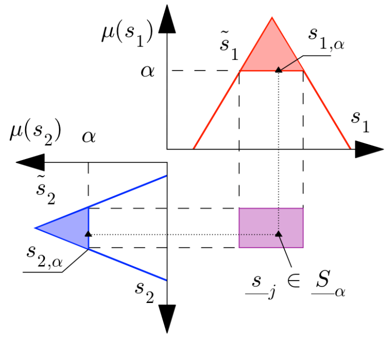
$$F(\tilde{s}, \underline{x}) = \{F_\alpha(\underline{x}); \mu(F_\alpha(\underline{x})) = [F_{\min,\alpha}(\underline{x}); F_{\max,\alpha}(\underline{x})], \mu(F_\alpha(\underline{x})) = \alpha \forall \alpha \in (0, 1]\} \tag{17}$$

with

$$F_{\min,\alpha}(\underline{x}) = \min[F(\underline{s}, \underline{x}) | \underline{s} \in \underline{S}_\alpha]$$
$$F_{\max,\alpha}(\underline{x}) = \max[F(\underline{s}, \underline{x}) | \underline{s} \in \underline{S}_\alpha]$$



(a) Fuzzy probability distribution $\tilde{F}(x)$



(b) Parameterization with $\tilde{s}_1$ and $\tilde{s}_2$.

Fig. 2 α-discretization of $F(\tilde{s}, \underline{x})$ in the presence of parameters $\tilde{s}_1$ and $\tilde{s}_2$.

2.3.3 Dynamic Models with Fuzzy Parameters

In the context of this work, the dynamic model describes the behavior of the rotor by means of a set of differential equations. The relationship between the inputs $\underline{x}$ and outputs $\underline{z}$ of an specific dynamic model M is characterized by $\mathbf{f}$, which represents the set of differential equations of the model in the Eq. (18)

$$M : \underline{z}(\tau) = \mathbf{f}(\underline{x}) \tag{18}$$

Therefore, the function $\mathbf{f}$ maps the inputs $\underline{x}$ onto the outputs $\underline{z}(\tau)$, thus $\underline{x} \to \underline{z}(\tau)$, where τ is the independent variable of the dynamic response that may represent time, frequency or spacial coordinates.

When considering the inputs of the model as fuzzy variables $\underline{\tilde{x}}$ or fuzzy functions $\underline{\tilde{x}}(\tau)$, the dynamic response of the system corresponds to the resulting fuzzy functions $\underline{\tilde{z}}(\tau)$. These fuzzy functions result of the mapping, thus $\underline{\tilde{x}} \to \underline{\tilde{z}}(\tau)$.

2.3.4 Fuzzy Dynamic Analysis

The fuzzy dynamic analysis is an appropriate method to map a fuzzy input vector $\underline{\tilde{x}}$ onto the output $\underline{\tilde{z}}(\tau)$ of a numerical model using the deterministic model given by Eq. (18). In the structural analysis, the combination of uncertainties modeled as fuzzy variables with the deterministic model based on Finite Element Method is denominated *Fuzzy Finite Element Method*. The fuzzy dynamic analysis is composed of two stages shown in the Fig. 3.

In the first stage, for computational purposes, the input vector that corresponds to the fuzzy parameter is discretized by means of α-level representation, presented in Eq. (12) and Fig. 1(b). Thus each element of the fuzzy parameters vector $\underline{\tilde{x}} = (\tilde{x}_1, \ldots, \tilde{x}_n)$ is considered as an interval $X_{i\alpha_k} = [x_{i\alpha_k l}, x_{i\alpha_k r}]$, where $\alpha_k \in (0, 1]$. Consequently, the sub-space $\underline{X}_{\alpha_k}$ is defined so that $\underline{X}_{\alpha_k} = (X_{1\alpha_k}, \ldots, X_{n\alpha_k})$, where $\underline{X}_{\alpha_k} \in \mathbb{R}^n$.

The second stage is related to solving an optimization problem. This optimization problem consists in finding the maximum and minimum value of the output, at each evaluated τ, for the mapping model $M : \underline{z} = f(\underline{x})$, thus:

$$z_{\alpha_k r} = \max_{\underline{x} \in \underline{X}_{\alpha_k}} \mathbf{f}(\underline{x}) \qquad z_{\alpha_k l} = \min_{\underline{x} \in \underline{X}_{\alpha_k}} \mathbf{f}(\underline{x}) \tag{19}$$

$z_{\alpha_k r}$ and $z_{\alpha_k l}$ correspond to the upper and lower bounds of the interval $z_{\alpha_k} = [z_{\alpha_k r}, z_{\alpha_k l}]$ in the α-level α_k. The set of discretized intervals $[z_{\alpha_k r}, z_{\alpha_k l}]$ for $\alpha_k \in (0, 1]$ composes the whole fuzzy resulting variable $\tilde{z}$.

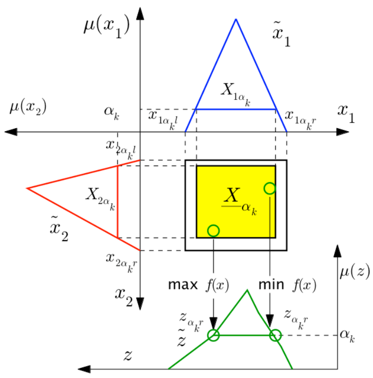


Fig. 3 α-Level optimization.

The fuzzy analysis of a transient time-domain system demands the solution of a large number of optimization problems regarding all α-level of interest for each considered time step. Each upper and lower bounds of the system analysis at a given time instant is obtained from the Differential Evolution optimization algorithm [10]. The output value of the transient analysis at the evaluated time-step constitutes the objective function. The inputs to this function are the uncertain parameters described previously as fuzzy or fuzzy random variables.

2.3.5 Fuzzy Stochastic Analysis

Uncertain structural analysis considers uncertain parameters on the numerical model (geometry or structural properties). The model is computed with the uncertain parameters to obtain the uncertain outputs, generally forces or displacements. The specific form of the uncertainties is determined by the uncertainty model. Essentially, an uncertainty can be modeled as a random, fuzzy or fuzzy-random variables or fields. In structural analysis, the combination of uncertainties modeled by using fuzzy randomness with the deterministic model based on Finite Element Method is denominated *Fuzzy Stochastic Finite Element Method*.

Fuzzy stochastic analysis is a straightforward computational method for processing uncertain data modeled by fuzzy random functions or variables. The aim of fuzzy stochastic analysis is to map fuzzy random parameters $\tilde{\underline{X}}$ onto the structural fuzzy random response $\tilde{\underline{Z}}(\tau)$, thereby, the following problem is to be solved for a crisp mapping model

$$\tilde{\underline{X}} \rightarrow \tilde{\underline{Z}}(\tau) \tag{20}$$

Fuzzy and real random variables are special cases of fuzzy random variables; this type of uncertainty can be also accounted for in Eq. (20). The basic concepts and definitions of the fuzzy stochastic analysis are presented in [7].

The fuzzy random parameters are modeled as fuzzy random variables, which represent a generalized uncertainty model i.e, high order uncertainty representation. The fuzziness of the fuzzy random variables $\tilde{\underline{X}}$ is described by means of fuzzy bunch parameters. Thus, by using the α-discretization the fuzzy bunch parameters $\tilde{\underline{s}}$ are discretized with the help of crisp α-level sets to obtain the corresponding intervals of each determined α-level.

The fuzzy stochastic analysis algorithm encompasses a fuzzy analysis and a stochastic analysis to cope with the fuzziness and randomness characteristics of the uncertainties (see Fig. 4). Both analyses are based on the deterministic model.

The purpose of fuzzy analysis is to map fuzzy input variables onto result variables; the result variables $\tilde{\underline{z}}$ are also fuzzy values. In this analysis, the α-level optimization as stated by [8] is applied, this strategy permits to map fuzzy variables without special properties. An extended formulation of this strategy was presented in Sec. 2.3.4.

In the stochastic analysis each element of fuzzy set $\tilde{s}$ determines an original real random variable of every fuzzy random variable and an original deterministic value of every fuzzy variable. These original variables are mapped onto the original function $Z(\tau)$ of fuzzy random result values $\tilde{Z}(\tau)$ by a stochastic analysis algorithm. The deterministic model is used to perform the stochastic analysis. Monte Carlo Simulation is an appropriated method to perform the stochastic analysis. A Spectral representation by Karhunen Loève expansion can be applied to model real random fields. The result of a Monte Carlo Simulation is a sample in one original function $Z(\tau)$ of the fuzzy random response $\tilde{Z}(\tau)$ on the membership level $\mu(Z(\tau)) = \alpha_k$. The assigned elements z_{α_k} of fuzzy bunch parameters (e.g. fuzzy mean or fuzzy variance) of the fuzzy random results are obtained through the statistic evaluation of the sample. The fuzzy stochastic analysis algorithm is presented in Fig. 4.

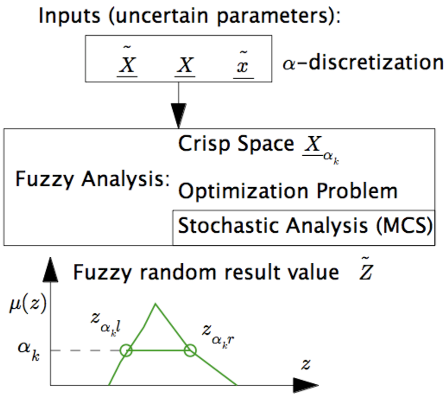


Fig. 4 Fuzzy stochastic analysis.

3 Reliability Analysis

Reliability analysis focuses on evaluating the likelihood that a system or component performs its intended function under uncertainty. Different mathematical frameworks have been proposed to estimate the probability or degree of failure. Probabilistic methods, such as Monte Carlo Simulation (MCS), the First-Order Reliability Method (FORM), and the Second-Order Reliability Method (SORM), provide a rigorous foundation when probabilistic information is available. Alternatively, fuzzy methods model vagueness and imprecision in system parameters, while interval methods rely on bounded representations when probabilistic data is scarce or unavailable. These approaches offer complementary perspectives, allowing reliability assessment under different types and levels of uncertainty.

3.1 Probabilistic Methods

The uncertain parameters can be modeled as random variables. These random variables are grouped in the random vector **c** associated to the joint probability density function. The mathematical model of the system is defined as $f(\mathbf{c})$ based on the system dynamics. It is worth to mention that system model represents the behavior of sytem output as a function of the input parameters. The kinematic reliability is quantified by the probability of the system output exceeding the maximum admissible limit, e_{max}, thus:

$$p_f = pr\{f(\mathbf{c}) > e_{max}\} \tag{21}$$

where $pr\{.\}$ represents the probability.

The failure probability is computed by using the Monte Carlo simulation method, first order reliability method (FORM), and the second order reliability method (SORM).

3.1.1 Monte Carlo Simulation Method

The MC method is composed of three steps: i) the uncertain parameters of the model ($f(\mathbf{c})$) are sampled to obtain a set of n_s random inputs; ii) the model outputs are computed for each random input. iii) the probability of failure is computed as:

$$p_f = \frac{n_f}{n_s} \tag{22}$$

where n_f is the number of samples that exceed e_{max}.

3.1.2 First Order Reliability Method (FORM)

This method is used to evaluate the failure probability of Eq. (21). The performance function $f(\mathbf{c})$ to evaluate the kinematic error is approximated by a first order Taylor expansion. Initially, the random vector $\mathbf{c}$ is transformed into random normal variables $\mathbf{u}$. Then, $f(\mathbf{c})$ is linearized at the most probable point (MPP) $\mathbf{u}^*$ and the reliability coefficient β is computed by solving the following optimization problem:

$$\begin{aligned} &\min_{\mathbf{u}} \beta = ||\mathbf{u}^T\mathbf{u}|| \\ &\text{subject to:} \\ &\delta\mathbf{p}(\mathbf{u}) = e_{max} \end{aligned} \tag{23}$$

where $||.||$ represents the magnitude of the vector. Finally, the probability of failure p_f is estimated according to the following expression: $p_f = \Phi[-\beta]$, were $\Phi[.]$ represents the standard normal cumulative distribution. The optimization problem to find the reliability coefficient β was solved by using the algorithm proposed by Rackwitz (1976). The numerical implementation of the algorithm demands the definition of an error tolerance, ϵ, to evaluate the convergence of β, and a small change in the coordinates, ι, to numerically evaluate the partial derivatives for the first-order Taylor expansion [11].

3.1.3 Second Order Reliability Method (SORM)

Differently from the FORM, this method aims at evaluating the reliability of system which performance functions is nonlinear. The performance function of the present contribution, $f(\mathbf{c})$. The Taylor series expansion of the nonlinear function $f(\mathbf{u})$ at the MMP $\mathbf{u}^*$ is defined as:

$$
\begin{aligned}
\delta\mathbf{p}(\mathbf{u}) =& \delta\mathbf{p}(u_1, u_2 \dots u_{n_j}) = \delta\mathbf{p}(u_1^*, u_2^* \dots u_{n_j}^*) \\
&+ \sum_{j=1}^{n_j} \frac{\partial \delta\mathbf{p}}{\partial u_j}(u_j - u_j^*) \\
&+ \frac{1}{2} \sum_{k=1}^{n_j} \sum_{j=1}^{n_j} \frac{\partial^2 \delta\mathbf{p}}{\partial u_k \partial u_j}(u_k - u_k^*)(u_j - u_j^*)
\end{aligned} \tag{24}
$$

where these derivatives are evaluated at the MMP $\mathbf{u}^*$. SORM ingores the terms higher than the second order terms. The probability of failure is computed by using the closed-form expression for the probability computation using theory of asymptotic approximation proposed by Breitung (1984), thus:

$$
p_f \approx \Phi(-\beta_{FORM}) \prod_{j=1}^{n_j - 1} (1 + \beta_{FORM}\kappa_j)^{-1/2} \tag{25}
$$

where κ_j represents the principal curvatures of the performance function at the MMP, and β_{FORM} is the reliability coefficient computed by the FORM.

3.2 Interval Methods

The sources of kinematic error (clearances or dimensional tolerances) can be modeled as a set of n intervals that can be grouped into the vector $\overline{\boldsymbol{c}} = (\boldsymbol{c}_l, \boldsymbol{c}_r)$ where $\boldsymbol{c}_l = \begin{bmatrix} c_{l1} & \dots & c_{ln} \end{bmatrix}$ is the lower limit and $\boldsymbol{c}_r = \begin{bmatrix} c_{r1} & \dots & c_{rn} \end{bmatrix}$ is upper limit. The output of the model system can be denoted as $f(\boldsymbol{c})$ that determines the errors on the output of the model.

The performance function $g(\boldsymbol{\lambda})$ is defined as the difference between a maximum allowable error (e_{max}) and the actual output of the model ($f(\boldsymbol{c})$), thus:

$$
g(\boldsymbol{\lambda}) = e_{max} - f(\boldsymbol{c}) \tag{26}
$$

The present work considers the errors as intervals $\overline{\boldsymbol{c}}$.

A two-dimensional performance function is considered $g(\overline{\boldsymbol{c}}) = g(\overline{c}_1, \overline{c}_2)$ to illustrate the present approach. A failure occurs if the kinematic error $f(\overline{\boldsymbol{c}})$ exceeds the admissible error, thus, $g(\overline{\boldsymbol{c}}) < 0$. Considering a two dimensional plane formed by the interval variables $\overline{c}_1) = (c_{1l}, c_{1r})$ and $\overline{c}_2 = (c_{2l}, c_{2r})$ as presented in Fig. 5(a). The curve $g(\boldsymbol{c})$ splits the rectangular area of the variables space into two regions: a safe area and a fail area. The total rectangular area is defined as $A_{total} = (c_{1r} - c_{1l})(c_{2r} - c_{2l})$; moreover, the possibility that a failure occurs (p_f) can be quantified as the quotient of the failure area A_{fail} and A_{total}, thus:

$$
p_f = \frac{A_{fail}}{A_{total}} \tag{27}
$$

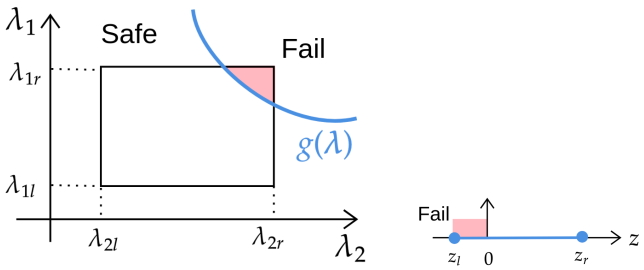
where p_f quantifies a percentual rate that internal error overpasses the admissible error limit $|e_{max}|$. If $f(\overline{c}) < |e_{max}|$ the failure index is zero; else, the expression of Eq. (27). As a result, the failure index is zero since the manipulator's desired operating state requires end-effector error to not exceed the maximum limit. Likewise, the reliability can be defined by R_{Int} as a percentual measure that indicates that the error will be contained within the maximum limit in the form:

$$R_{Int} = 1 - p_f \tag{28}$$

The more reliability implies that R_{Int} is close to 1. Moreover, the failure area (A_{total}) can be determined by:

$$A_{fail} = \int\int_{g(\boldsymbol{c})<0} g(c_1, c_2)d\,c_2 d\,c_1 \tag{29}$$

Therefore, the integral of Eq. (29) should be numerically solved to estimate the failure possibility of failure. It is worth mentioning that the performance function $g(\boldsymbol{c})$ depends on the nonlinear kinematic model of the manipulator that defines the positioning error.



(a) Variable space.

(b) Interval performance variable $\overline{\boldsymbol{z}}$.

Fig. 5 Interval Reliability.

Considering a n-dimensional case for the assessment of the reliability with n interval sources of errors defined as $\overline{\boldsymbol{c}} = \begin{bmatrix}\overline{c}_1 & \dots & \overline{c}_n\end{bmatrix}$. The new interval variable $\overline{\mathbf{z}} = (z_l, z_r)$ is defined and it depends on the interval performance function $g(\overline{\boldsymbol{c}})$. Based on the definition of the performance function, $\overline{\mathbf{z}} = e_{max} - e_T(\overline{\boldsymbol{c}})$. It is worth to saying that the assessment of $\overline{\mathbf{z}}$ depends on the evaluation of the interval error $e_T(\overline{\boldsymbol{c}}) = (e_l, e_r)$.

The evaluation of e_r and e_l requires the resolution of an optimization problem in order to ascertain the maximum and minimum limits of the interval error generated by the interval parameters $\overline{\boldsymbol{c}} \in (\boldsymbol{c}_l, \boldsymbol{c}_r)$ in the form.

$$e_r = \max_{\boldsymbol{c}\in\overline{\boldsymbol{c}}} e_T(\boldsymbol{c}) \qquad\qquad e_l = \min_{\boldsymbol{c}\in\overline{\boldsymbol{c}}} e_T(\boldsymbol{c}) \tag{30}$$

However, a failure can occur in the case that interval error, e_T, exceeds an admissible limit e_{max}. The failure measures the chance that the interval error will exceed the maximum error limit, e_{max}. According to Eq. (5) a failure occurs when $\overline{\mathbf{z}} < 0$ as presented in Fig. 5(b). The failure area A_{fail} and the total area A_{total} of Eq. (29) corresponds to the expressions of Eq. (30) where $A_{fail} = e_r - e_{max}$ and $A_{total} = e_r$. Thus, the interval reliability is computed with the following expressions:

$$R_{Int} = 1 - \frac{e_r - e_{max}}{e_r} \tag{31}$$

3.3 Fuzzy Methods

This method aims at computing the fuzzy reliability index to assess the kinematic accuracy of a system subject to an uncertainties. The proposed approach is based on the fuzzy uncertainty theory.

The possibility that the fuzzy output, $\tilde{\mathbf{f}}$, exceeds the maximum limit e_{max} is quantified by the fuzzy reliability, α_f, thus:

$$\alpha_f = \mu\{\tilde{\mathbf{f}} > e_{max}\} \tag{32}$$

where $\mu\{.\}$ represents the possibility that is quantified by the membership function value α_f with $0 \leq \alpha_f \leq 1$.

The fuzzy reliability aims at determining the possibility of failure by examining the fuzzy output. Figure 3.3 shows that a failure is produced when $\tilde{f} > e_{max}$. For this condition, the possibility of failure is given by α_f.

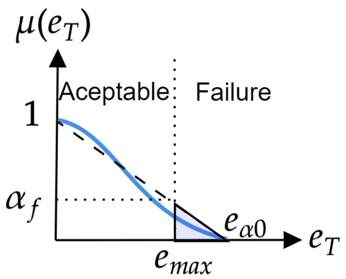


Fig. 6 Fuzzy output, $\tilde{\mathbf{f}}$, and linear approximation to estimate α_f.

For this particular application, the membership function of the fuzzy output $\mu(\tilde{\mathbf{f}})$ is approximated as a linear function in order to estimate the failure possibility considering the maximum position error $e_{\alpha 0}$ only. This linear approximation is the simplest form to compute the possibility $\mu\{.\}$ of Eq. (32) to obtain the failure possibility α_f; however, high order functions could be used to enhance the approximation of α_f. If the fuzzy error exceeds the maximum limit, i.e. if the fuzzy output surpasses e_{max} and it attains the failure region (see Fig. 3.3), the following expression computes the linear estimation of the failure possibility:

$$\alpha_f = \frac{e_{\alpha_0} - e_{max}}{e_{\alpha_0}} \tag{33}$$

otherwise, if fuzzy error is less than e_{max}, the fuzzy error is confined in the aceptable region (see Fig. 3.3), then $\alpha_f = 0$. Moreover, e_{α_0} which is the uncertain fuzzy error $\tilde{\mathbf{f}}$ evaluated at the α-level $\alpha_k = 0$ is obtained by solving the following maximization:

$$e_{\alpha_0} = \max_{\boldsymbol{\lambda}_{\alpha_0}} e(\boldsymbol{c}) \tag{34}$$

The optimization problem of Eq. (34) could be solved to compute the failure possibility α_f of Eq. (33); this optimization was solved by using the Differential Evolution (DE) algorithm [10].

The failure possibility, α_f, is bounded as follows:

$$0 \leq \alpha_f \leq 1 \tag{35}$$

where the desired performance of a manipulator consists of minimizing the failure possibility. It is worth mentioning that the failure possibility indicates the percentual rate of the maximum fuzzy error $\tilde{\mathbf{e}}_T$ surpassing the maximum admissible error e_{max}, e.g. $\alpha_f = 0.45$ indicates that the maximum fuzzy error overpasses 45% of the maximum admissible error e_{max}.

4 Robust Design

The theoretical basis of reliability-based optimization (RBO) was introduced by Moses et al. (1967) [13]. The robust-based optimization (RBO) is presented in Eqs. (36) to (39) according to [14] where $f(.)$ represents the objective function, $\mathbf{x}_d$ is the vector of the determinist design variables, $\mathbf{x}_u(\theta)$ are the uncertain inputs modeled as random variables, and θ represents a stochastic process. This optimization problem is subject to the n_g reliability constraints of Eq. (37) where $P(.)$ denotes the joint probability of the solution being feasible for every j constraint by considering the uncertainties $\mathbf{x}_u(\theta)$, i.e., the reliability express the probability of obtaining a feasible solution under the uncertain inputs. $R \in [0, 1]$ is the desired reliability for all $j$ht constraints. The more reliable solution implies that R is close to one. $h_k(\mathbf{x}_d)$ of Eq. (38) denotes the deterministic inequality constraints, and Eq. (39) defines the constraints of the design variables.

$$\min_{\mathbf{x}_d} \quad f(\mathbf{x}_d, \mathbf{x}_u(\theta)) \tag{36}$$

$$\text{subject to} \quad P(g_j(\mathbf{x}_d, \mathbf{x}_u(\theta)) \leq 0) \geq R, \quad j = 1, \ldots, n_g \tag{37}$$

$$h_k(\mathbf{x}_d) \leq 0, \quad k = 1, \ldots, n_k \tag{38}$$

$$\mathbf{x}_d^{(l)} \leq \mathbf{x}_d \leq \mathbf{x}_d^{(u)} \tag{39}$$

Let us consider the optimization problem with two inequality constraints shown in Fig. 7. The deterministic optimal solution without uncertainty ($\mathbf{x}_u = 0$) is obtained by solving Eq. (36) and neglecting the reliability constraints of Eq. 37); one can observe that deterministic optimum is located at the intersection of the constraints. On the other hand, the reliability-based optimization problem considers the effect of uncertain inputs; these uncertain inputs produce variation around the optimal solution making this solution infeasible in many instances. Consequently, the reliable solution will be penalized by considering the reliability constraints of Eq. (37). A reliable solution will imply a small probability of obtaining an infeasible solution. The feasible solution with the desired reliability R will guarantee that the probability of obtaining an infeasible solution due to uncertainties is defined as $(1 - R)$.

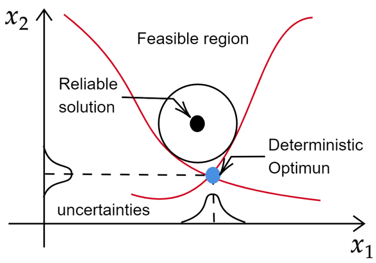


Fig. 7 Reliability-based optimization procedure, adapted from [14].

5 Applications

5.1 Structures

5.1.1 Uncertainty Analysis

Uncertainty analysis of flexible rotors has been studied by applying stochastic approaches based on the stochastic finite element method [15]. Didier et al. [16] quantified the uncertainties effects in the response of flexible rotors based on the Polynomial Chaos theory. Koroishi et al. [15] represented the uncertainties in the rotor parameters by using Gaussian homogeneous stochastic fields discretized by Karhunen-Loève expansion; the dynamic response of the system with random parameters was characterized through Hypercube Latin sampling and Monte Carlo simulation. Lara-Molina et al. [17] used the fuzzy stochastic finite element method to quantify the effects of high order random parameters on the response of a rotating machine. The analyses of the effect of uncertain parameters on the dynamic behavior of a flexible rotor containing two rigid discs and supported by two fluid film bearings has been analyzed considering the inherent uncertainties of the bearings' parameters (i.e. the oil viscosity as a function of the oil temperature, and the radial clearance) modeled by using a fuzzy dynamic analysis [18].

For comparison purposes, the Monte Carlo simulation combined with Latin Hypercube sampling is used to generate the envelope of responses of the stochastic rotor system. The choice of this stochastic solver is justified by the fact that Monte Carlo simulation has been successfully used as a reference stochastic solver to evaluate the variability of the dynamic responses [19].

Cavalini et al. [20] analyzes the uncertainties that affect the load capability of a 4-pad tilting-pad journal bearing using fuzzy logic techniques. Moreover, Cavalini Jr et al. [21] assess the uncertainties affecting the dynamic behavior of a flexible rotor containing three rigid discs and supported by two cylindrical fluid film bearings using probabilisitc approaches and also fuzzy methods. Lara-Molina et al. [22] evaluate the effects of uncertainties on the load capacity of a four-pad tilting-pad journal bearing, in which the pad radius, oil viscosity, and radial clearance are considered as uncertain information; thus, the hydrodynamic supporting forces at the bearing pads are obtained by solving the Reynolds equation. In this case, the uncertain parameters are modeled as fuzzy type-2 variables. Lara-Molina et al. [23] analyzes the sensitivity of the structural response of flexible rotors subjected to uncertain interval parameters.

Zuffi et al. [24] analyze the effect of uncertain parameters on a mechanical system that represents the phenomenon of near-field acoustic levitation, whose dynamic behavior is described by the Reynolds equation. Teixeira et al. [25] aimed at comparing the so-called Bingham, modified Bouc-Wen (BW), and hysteretic models dedicated to MR actuators; additionally, uncertainty and sensitivity analyses based on the interval approach were applied on the updated MR models aiming to determine the working envelopes associated with the most important parameters of the models. Barbosa et al. [26] applied uncertainty and sensitivity analyses to the SHBT-FE model of a composite hollow shaft. Silva et al. [27] offered a comprehensive evaluation of the composite material hollow shaft's frequency response functions by considering a more extensive set of uncertain parameters as compared to previous studies.

5.1.2 Robust Design

In general, various reliability strategies have been presented in the literature to include uncertainties into the optimal design of manipulators. The reliability-based optimization (RBO) is the most employed approach of the above mentioned optimal design methods. According to Du and Chen [28], RBO aims to obtain high reliability by minimizing the contribution of uncertainties on the objective functions, imposing constraint levels defined in terms of reliability. Traditionally, Monte Carlo simulation has been used to evaluate the probability value, despite the high computational cost associated with this approach [28]. As an alternative, RBO without using simulation methods has been proposed [29]. In general, in the aforementioned strategies, the random variables are transformed from physical to standard space. As a consequence, a new optimization is derived for computing the most significant failure probability that is at the same time assigned to an imposed value that satisfies the design requirements [29]. According to methods to the failure probability computation, the RBO approached are split into four groups [29]: *i*) single-loop methods, *ii*) decoupled methods, *iii*) double-loop methods, and *iv*) simulation methods.

There are several works in literature dedicated to the stochastic analysis of dynamic vibration absorbers, in which the effect of uncertainties is analyzed. Borges et al. [30] studied a robust optimization method applied to nonlinear dynamic absorbers.

Chakraborty and Roy [31] showed the reliability based optimum design by considering bounded uncertain parameters. On the other hand, several studies have applied robust optimization to minimize the effects of uncertainties while the optimal performance is maximized. Silva et al. [32] proposed a robust optimization based on a fuzzy logic approach, in which the uncertain parameters of linear and nonlinear dynamic vibration absorber were considered. It is worth mentioning that robust optimization has been properly applied in various dynamic systems, including dynamic vibration absorbers [30, 33].

6 Comparison of Uncertainty and Reliability Analysis Methods

The different approaches to uncertainty and reliability analysis—probabilistic, fuzzy, and interval-based—provide complementary tools for modeling and propagating uncertainty in engineering systems. Their applicability depends on the type of uncertainty to be represented, the amount of available data, and the computational resources required. This section presents a comparative discussion of these methods, emphasizing their conceptual differences, advantages, and limitations.

6.1 Comparative Discussion

Probabilistic methods rely on the statistical characterization of uncertain parameters through probability distributions. They provide detailed quantitative information about system behavior and allow direct estimation of reliability indices such as the probability of failure. Techniques such as Monte Carlo Simulation (MCS), First-Order Reliability Method (FORM), and Second-Order Reliability Method (SORM) are well established and widely used in engineering applications. However, their accuracy depends on the adequacy of the assumed probability distributions, and their computational cost can be significant for complex models or rare-event probabilities.

Fuzzy methods are suitable when uncertainty arises from vagueness, imprecision, or linguistic assessments rather than randomness. In these methods, uncertain quantities are represented by fuzzy numbers or membership functions that express degrees of possibility rather than probability. Fuzzy reliability analysis can effectively capture epistemic uncertainty, but it provides qualitative or semi-quantitative results that are not directly comparable to probabilistic measures. Moreover, the definition of membership functions often involves subjective judgment.

Interval methods offer a conservative and data-independent framework for uncertainty representation. Instead of assuming a distribution or a membership function, uncertain parameters are expressed as bounded intervals within which the true values are expected to lie. These methods are computationally efficient and straightforward to implement, making them particularly useful in early design stages or when information is scarce. However, the results tend to be over-conservative, and the lack of probabilistic interpretation may limit their use in risk-based decision-making.

6.2 Summary of Characteristics

Table 1 summarizes the main characteristics of probabilistic, fuzzy, and interval methods for uncertainty and reliability analysis.

Table 1 Comparison of probabilistic, fuzzy, and interval methods for uncertainty and reliability analysis.

Aspect	**Probabilistic Methods**	**Fuzzy Methods**	**Interval Methods**
Type of Uncertainty	Aleatory (randomness)	Epistemic (imprecision)	Epistemic (bounded uncertainty)
Input Data Required	Statistical samples or PDFs	Expert knowledge, linguistic data	Parameter bounds only
Output Nature	Probabilities, reliability indices	Possibility distributions	Upper/lower bounds on response
Computational Cost	High (especially for MCS)	Moderate	Low
Interpretability	Quantitative and rigorous	Qualitative or semi-quantitative	Conservative and bounded
Typical Applications	Structural reliability, risk analysis, performance assessment	Expert systems, human reliability, vague input data	Preliminary design, limited-data modeling

Probabilistic methods rely on statistical modeling and data availability.

[1]Fuzzy methods are suitable when data is imprecise or linguistic.

[2]Interval methods are conservative and require only parameter bounds.

6.3 Remarks

The comparative analysis demonstrates that each framework has specific strengths and limitations. Probabilistic methods remain the most rigorous and informative when sufficient statistical information is available. Fuzzy and interval approaches are more appropriate when uncertainty arises from lack of knowledge or incomplete data. Hybrid methodologies that combine probabilistic and non-probabilistic representations offer a promising direction to balance accuracy, interpretability, and computational efficiency in real-world engineering problems.

6.4 Mechanisms

6.4.1 Uncertainty Analysis

Multi body mechanisms are unavoidably subject to uncertainties. The main sources of uncertainties include various aspects such as manufacturing limitations and assembling tolerances of the mechanical parts, noise in the sensors, and unmodeled dynamics in the control system. Furthermore, in several applications, the robots operate with different values of payload to perform a specific task (e.g. pick and place robots).

Despite uncertainties, the parallel robots should be able to execute diverse tasks with high accuracy and repeatability which requires high reliability (e.g. robots used in medical applications). Therefore it is necessary to analyze the effects of uncertain parameters on the dynamic response in order to observe the behavior of the parallel robots under these conditions.

Several methodologies have been used to analyze uncertainties in robot manipulators. The stochastic approach has been widely applied to study the effects of uncertain parameters on the behavior of robot manipulators. In agreement with this approach, the effect of tolerances associated with the manipulator parameters on the reliability was studied [34]. Moreover, Polynomial Chaos Theory was applied to study the effect of uncertain inertia and payload on SCARA robot dynamics [35]. Lara-Molina [36] presented an approach to obtain the dynamic model of flexible-link manipulators based on the stochastic finite element method. Lara-Molina et al. [37] developed a method for analyzing the effect of these uncertainties on the kinematic performance of the mechanism by examining the kinematic performance atlases.

Interval analysis has been also applied to study the uncertainties aiming at ensuring the reliability of robot manipulators [38]. Additionally, an approach based on fuzzy dynamic analysis has been applied to study uncertain parameters in a robot manipulator [39]. The aforementioned approaches are suitable when the stochastic process that governs the uncertainty is unknown; thus uncertain parameters are modeled by means of fuzzy variables. Lara-Molina et al. [40] caried out the uncerainty analysis of a Stewart-Gough platform to analyze the sentisivity of dynamics outputs of the numerical model subjected to uncertain paramenters parameters. Lara-Molina and Dumur [41] proposed an alternative approach based on a fuzzy-interval analysis to assess how dimensional tolerances affect the kinematics of parallel manipulators, its novelty consists of modeling and analyzing the dimensional tolerances as fuzzy-intervals based on fuzzy theory and interval sensitivity methods.

6.4.2 Robust Optimization

The optimal design has widely used multi-objective optimization to maximize the performance index of the parallel manipulators; nevertheless, the multi-objective design of manipulators has not taken into account the unavoidable effect of the uncertainties. Thus, it is necessary to define a robustness criterion that measures how the uncertainties jeopardize the performance of parallel manipulators. Furthermore, the robustness of the manipulators for the optimal design has not been considered as a global criterion; i.e., the robustness was computed for a single pose [42, 43]. The robust design based on multi-objective optimization has been applied to flexible structures, mechanisms, and mechanical systems. Nevertheless, research studies reported in the literature have not explored the optimal design of robotic manipulators subject to uncertainties. In this context, the novelty of the present contribution consists of proposing a novel optimal design procedure of parallel manipulators that includes the detrimental effect of the uncertainties on the performance by considering a robust criterion within the multi-objective design optimization. The proposed robust optimization is composed of three stages: *i*) modeling of the uncertainties that affect performance indices; *ii*) definition of the performance indices and robustness criterion as global criteria that evaluates these proprieties over a region of the workspace; *iii*) formulation of robust optimization as a multi-objective optimization problem subject to nonlinear constraints.

Several numerical methods have been developed to enhance computational cost and accuracy of robust optimal design applied to optimize mechanical structures subject to uncertain operational conditions. A Hierarchical Model Updating Strategy was implemented for Finite Element model updating to establish an accurate computational model [44]. A synchronous modeling concept is proposed, the purpose of which is to realize the transformation from single-objective reliability design to multiple objectives reliability design [45]. In the same direction, multilevel nested models were developed to effectively perform the reliability-based design optimization of the assembly relationship [46]. These developments in reliability computation and reliability-based design optimization permitted to carry out of further engineering applications. Li et al. (2022) [47] developed a method to identify the modal parameters of damped oscillation signal in a power system; this application demonstrated the potential benefits of the numerical methods to improve the efficiency of reliability computation. Moreover, Fei et al. (2020) [48] evaluated the motion reliability of flexible mechanism; they demonstrated that the enhancement of the network learning model with an intelligent operator ameliorated the reliability computation in terms of simulation efficiency to design procedures. Martins et al. [49] analyzed the effect of uncertain parameters on the frequency-domain behavior of a dynamic vibration absorber for the robust optimization that improves its performance based on uncertainty and sensitivity analyses. Lara-Molina and Gonçalves [50] addressed the optimal design of flexible link manipulators to optimize the dynamic performance subject to the effect of the uncertainties quantified by a reliability index. Lara-Molina and Dumur [51] developed a novel robust optimal design for parallel manipulators to optimize the performance indices subject to the unavoidable effect of the uncertainties.

6.4.3 Reliability Assessment

The kinematic reliability of manipulators determines the probability of obtaining positioning errors within acceptable limits. The kinematic reliability has recently emerged as alternative criteria to evaluate the effects of uncertainties in manipulators [52]. Kim et al (2010) evaluates the kinematic reliability of manipulators using the advanced first-order second moment (AFOSM) method. Pandey and Zhang (2012) used the fractional moments to efficiently compute the kinematic reliability such that the positioning error remains within aceptable limits. Cui et al. (2015) computed the kinematic reliability using the Monte Carlo simulation method and they evaluated three error sensitivity criteria based on the singular value decomposition of the error translation matrix. Zhan et al. (2018) proposed an hybrid method based on the first order second moment to evaluate the uncertainties of a planar parallel manipulator modeled as random and interval varaiables. Xu (2018) studied the influence of each error source on the kinematic reliability of a delta parallel manipulator. Zhang and Han (2020) develops an efficient reliability analysis method to account for random dimensions and joint angles of robotic mechanisms. Moreover, further developments about the kinematic reliability have recently been reported in literature considering improvements in the computational methods [57], application on industrial robots [58], and applications on planar mechanisms [59]. The research works mentioned above evaluated the reliability as a local property, i.e. the kinematic reliability was assessed at a specific pose of the manipulator. Nevertheless, it is necessary to compute the kinematic reliability as a global criteria. Lara-Molina and Dumur [60] developed a performance criterion applied to robotic manipulators based on kinematic reliability based on the kinetostatic performance criteria to quantify the effect of errors in the manipulators.

As an alternative to the probabilistic methods, a non-probabilistic approach based on the Lagrange multipliers and Taylor expansion has been proposed [61] to evaluate the kinematic reliability of the mechanism with clearances. Moreover, a time-dependent reliability index was described based on a combination of the nonprobabilistic interval process and first-passage theories. [62]. Lara-Molina and Gonçalves [63] proposed a novel method based on an interval approach to evaluate the kinematic reliability of manipulators that quantifies the probability of positioning errors that fall within allowable boundaries.

The methods based on a probabilistic approach computes an indirect measure of the effect of uncertainties based on the probability of failure. The probability of failure expresses how likely the positioning error is greater than a maximum limit; it depends on the probability distribution imposed on the uncertain parameters. Nevertheless, in several applications, it is not possible to find the probability distribution of the uncertain parameters based on experimental measures [61].

Non-probabilistic methods based on fuzzy theory have been used to analyze the reliability of robotic manipulators [64]. Moreover, Non-probabilistic methods based on interval theory have widely been applied in several engineering areas for uncertainty quantification [65] and reliability analysis [66]. Moreover, interval theory has been applied to analyze the effect of dimensional tolerances of mechanism [67], positioning accuracy [68], and reliability analysis [69]. Nevertheless, the non-probabilistic method based on interval analysis has yet to be explored to analyze the kinematic reliability of robotic manipulators.

7 Conclusions

This manuscript presented a comprehensive overview of uncertainty and reliability analysis methods, encompassing probabilistic, fuzzy, and interval-based frameworks. The study discussed the theoretical foundations and main characteristics of each approach, emphasizing their applicability to engineering problems where model parameters and input data are subject to different forms of uncertainty.

Probabilistic methods, such as the Monte Carlo Simulation (MCS), First-Order Reliability Method (FORM), and Second-Order Reliability Method (SORM), provide a rigorous mathematical basis for modeling randomness when sufficient statistical information is available. However, in many practical situations, the lack of reliable probabilistic data justifies the use of non-probabilistic approaches. In this context, fuzzy methods offer a flexible means to capture epistemic uncertainty through linguistic or membership functions, while interval methods provide conservative but computationally efficient estimates when only parameter bounds are known.

Overall, the comparative analysis highlights that no single method is universally superior; rather, the selection of an appropriate uncertainty representation depends on the nature and quality of the available information, the complexity of the model, and the desired level of confidence in the results. The integration of probabilistic and non-probabilistic frameworks emerges as a promising direction to enhance the robustness and interpretability of engineering analyses under uncertainty.

Future research should focus on hybrid uncertainty modeling strategies, surrogate-based propagation techniques, and the incorporation of data-driven approaches to reduce computational cost and improve model fidelity. Such developments will strengthen the role of uncertainty and reliability analysis as essential tools for predictive modeling and robust decision-making in complex engineering systems.

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Declaração de disponibilidade de dados da pesquisa

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Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, noethical approval is required.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

References

[1] Soize, C.: Uncertainty Quantification vol. 23. Springer, ??? (2017)

[2] Bi, S., He, K., Zhao, Y., Moens, D., Beer, M., Zhang, J.: Towards the nasa uq challenge 2019: Systematically forward and inverse approaches for uncertainty propagation and quantification. Mechanical Systems and Signal Processing **165**, 108387 (2022)

[3] Wong, S.K.M., Wang, L., Yao, Y.Y.: On modeling uncertainty with interval structures. Computational Intelligence **11**(2), 406–426 (1995)

[4] Ratz, D.: Box-splitting strategies for the interval gauss-seidel step in a global optimization method. Computing **53**(3), 337–353 (1994)

[5] Hickey, T., Ju, Q., Van Emden, M.H.: Interval arithmetic: From principles to implementation. Journal of the ACM (JACM) **48**(5), 1038–1068 (2001)

[6] Zadeh, L.: Fuzzy sets. Information and Control **8**, 338–353 (1965)

[7] Möller, B., Beer, M.: Fuzzy Randomness, Uncertainty in Civil Engineering and Computational Mechanics. Springer, ??? (2004)

[8] Möller, B., Graf, W., Beer, M.: Fuzzy structutal analysis using α-level optimizaton. Computational Mechanics **26**, 547–565 (2000)

[9] Puri, M.L., Ralescu, D.A.: Fuzzy random variables. Journal of Mathematical Analysis and Applications **114**(2), 409–422 (1986)

[10] Price, K.V., Storn, R.M., Lampinen, J.A.: Differential Evolution a Practical Approach to Global Optimization. Springer, ??? (2005)

[11] Rackwitz, R.: Practical probabilistic approach to design. bulletin, 112. Comite European du Beton (1976)

[12] Breitung, K.: Asymptotic approximations for multinormal integrals. Journal of Engineering Mechanics **110**(3), 357–366 (1984)

[13] Moses, F., Kinser, D.E.: Optimum structural design with failure probability constraints. AIAA journal **5**(6), 1152–1158 (1967)

[14] Deb, K., Gupta, S., Daum, D., Branke, J., Mall, A.K., Padmanabhan, D.: Reliability-based optimization using evolutionary algorithms. IEEE transactions on evolutionary computation **13**(5), 1054–1074 (2009)

[15] Koroishi, E.H., Cavalini Jr, A.A., Lima, A.M., Steffen Jr, V.: Stochastic modeling of flexible rotors. Journal of the Brazilian Society of Mechanical Sciences and Engineering **34**, 574–583 (2012)

[16] Didier, J., Faverjon, B., Sinou, J.-J.: Analysing the dynamic response of a rotor system under uncertain parameters by polynomial chaos expansion. Journal of Vibration and Control **18**(5), 712–732 (2012)

[17] Lara-Molina, F.A., Koroishi, E.H., Steffen Jr, V.: Uncertainty analysis of flexible rotors considering fuzzy parameters and fuzzy-random parameters. Latin American Journal of Solids and Structures **12**(10), 1807–1823 (2015)

[18] Cavalini Jr, A.A., Lara-Molina, F.A., Sales, T.d.P., Koroishi, E.H., Steffen Jr, V.: Uncertainty analysis of a flexible rotor supported by fluid film bearings. Latin American Journal of Solids and Structures **12**(8), 1487–1504 (2015)

[19] Sampaio, R., Cataldo, E.: Comparing two strategies to model uncertainties in structural dynamics. Shock and Vibration **17**(2), 171–186 (2010)

[20] Cavalini, A., Dourado, A., Lara-Molina, F., Steffen, V.: Uncertainty analysis of a tilting-pad journal bearing using fuzzy logic techniques. Journal of Vibration and Acoustics **138**(6), 061016 (2016)

[21] Cavalini Jr, A.A., Silva, A.D.G., Lara-Molina, F.A., Steffen Jr, V.: Dynamic analysis of a flexible rotor supported by hydrodynamic bearings with uncertain parameters. Meccanica **52**(11), 2931–2943 (2017)

[22] Lara-Molina, F., Cavalini Jr, A., Steffen Jr, V., Cabrera, N.: Tilting-pad journal bearing subjected to fuzzy type-2 uncertain parameters. Journal of Vibration and Acoustics **141**(6), 061008 (2019)

[23] Lara-Molina, F.A., Cavalini Jr, A.A., Koroishi, E.H., Steffen Jr, V.: Sensitivity analysis of flexible rotor subjected to interval uncertainties. Latin American Journal of Solids and Structures **16**(04), 188 (2019)

[24] Zuffi, G.A., Lobato, F.S., Lara–Molina, F.A., Junior, A.A.C., Junior, V.S.: Análise de sensibilidade em um sistema de levitação acústica de campo próximo sujeito a incertezas. Revista Mundi Engenharia, Tecnologia e Gestão (ISSN: 2525-4782) **4**(5) (2019)

[25] Teixeira, P.C.F., Saad, N.d.S., Lara-Molina, F.A., Cavalini Jr, A.A., Steffen Jr, V.: Evaluation of parametric models dedicated to a magnetorheological actuator including uncertainty and sensitivity analyses. ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part B: Mechanical Engineering **5**(4), 041003 (2019)

[26] Barbosa, P.C.d.F., Lara-Molina, F.A., Silva, I.B., Cavalini Jr, A.A., Steffen Jr, V.: Uncertain and sensitivity analyses of a composite shaft. Meccanica **55**(1), 35–48 (2020)

[27] Silva, L.S., Cavallini Jr, A.A., Lara-Molina, F.A., Steffen Jr, V.: Interval uncertainty and sensitivity analysis of the dynamic behavior of a composite material hollow shaft. Journal of the Brazilian Society of Mechanical Sciences and Engineering **47**(8), 403 (2025)

[28] Du, X., Chen, W.: Sequential optimization and reliability assessment method for efficient probabilistic design. J. Mech. Des. **126**(2), 225–233 (2004)

[29] Deb, K., Gupta, S., Daum, D., Branke, J., Mall, A.K., Padmanabhan, D.: Reliability-based optimization using evolutionary algorithms. IEEE transactions on evolutionary computation **13**(5), 1054–1074 (2009)

[30] Borges, R., Lima, A., Steffen Jr, V.: Robust optimal design of a nonlinear dynamic

vibration absorber combining sensitivity analysis. Shock and Vibration **17**(4-5), 507–520 (2010)

[31] Chakraborty, S., Roy, B.K.: Reliability based optimum design of tuned mass damper in seismic vibration control of structures with bounded uncertain parameters. Probabilistic Engineering Mechanics **26**(2), 215–221 (2011)

[32] Silva, A., Cavalini Jr, A., Steffen Jr, V.: Fuzzy robust design of dynamic vibration absorbers. Shock and Vibration **2016**(1), 2081518 (2016)

[33] Zang, C., Friswell, M., Mottershead, J.: A review of robust optimal design and its application in dynamics. Computers & structures **83**(4-5), 315–326 (2005)

[34] Pandey, M.D., Zhang, X.: System reliability analysis of the robotic manipulator with random joint clearances. Mechanism and Machine Theory **58**, 137–152 (2012)

[35] Voglewede, P., Smith, A.H., Monti, A.: Dynamic performance of a scara robot manipulator with uncertainty using polynomial chaos theory. IEEE Transactions on Robotics **25**(1), 206–210 (2009)

[36] Lara-Molina, F.A.: Modeling of flexible-link manipulators under uncertain parameters based on stochastic finite element method. Journal of Mechanisms and Robotics **14**(6), 061002 (2022)

[37] Lara-Molina, F., Koroishi, E., Steffen Jr, V., Martins, L.: Kinematic performance of planar 5r symmetrical parallel mechanism subjected to clearances and uncertainties. Journal of the Brazilian Society of Mechanical Sciences and Engineering **40**(4), 189 (2018)

[38] Merlet, J.-P.: Interval analysis and reliability in robotics. International Journal of Reliability and Safety **3**(1-3), 104–130 (2009)

[39] Lara-Molina, F., Koroishi, E., Steffen, V.: Uncertainty analysis of a two-link robot manipulator under fuzzy parameters. In: 2014 Joint Conference on Robotics: SBR-LARS Robotics Symposium and Robocontrol, pp. 1–6 (2014). IEEE

[40] Lara-Molina, F.A., Koroishi, E.H., Dumur, D., Steffen Jr, V.: Stochastic analysis of a 6-dof fully parallel robot under uncertain parameters. IFAC-PapersOnLine **48**(19), 214–219 (2015)

[41] Lara-Molina, F.A., Dumur, D.: Fuzzy-interval approach and sensitivity analysis to assess dimensional tolerances of parallel manipulators. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science **238**(1), 108–122 (2024)

[42] Caro, S.p., Bennis, F., Wenger, P.: Tolerance synthesis of mechanisms: a robust

design approach. J. Mech. Des. **127**(1), 86–94 (2005)

[43] Jørgensen, T.B., Wolniakowski, A., Petersen, H.G., Debrabant, K., Krüger, N.: Robust optimization with applications to design of context specific robot solutions. Robotics and Computer-Integrated Manufacturing **53**, 162–177 (2018)

[44] FEI, C., LIU, H., PATRICIA LIEM, R., CHOY, Y., HAN, L.: Hierarchical model updating strategy of complex assembled structures with uncorrelated dynamic modes. Chinese Journal of Aeronautics **35**(3), 281–296 (2022) https://doi.org/10.1016/j.cja.2021.03.023

[45] Fei, C.-W., Li, H., Lu, C., Han, L., Keshtegar, B., Taylan, O.: Vectorial surrogate modeling method for multi-objective reliability design. Applied Mathematical Modelling **109**, 1–20 (2022) https://doi.org/10.1016/j.apm.2022.03.033

[46] Fei, C.-W., Li, H., Liu, H.-T., Lu, C., Keshtegar, B., An, L.-Q.: Multilevel nested reliability-based design optimization with hybrid intelligent regression for operating assembly relationship. Aerospace Science and Technology **103**, 105906 (2020) https://doi.org/10.1016/j.ast.2020.105906

[47] Li, H., Bu, S., Wen, J.-R., Fei, C.-W.: Synthetical modal parameters identification method of damped oscillation signals in power system. Applied Sciences **12**(9) (2022)

[48] Fei, C.-W., Li, H., Liu, H.-T., Lu, C., An, L.-Q., Han, L., Zhao, Y.-J.: Enhanced network learning model with intelligent operator for the motion reliability evaluation of flexible mechanism. Aerospace Science and Technology **107**, 106342 (2020) https://doi.org/10.1016/j.ast.2020.106342

[49] Martins, L.A., Lara-Molina, F.A., Koroishi, E.H., Cavalini Jr, A.A.: Optimal design of a dynamic vibration absorber with uncertainties. Journal of Vibration Engineering & Technologies **8**(1), 133–140 (2020)

[50] Lara-Molina, F.A., Gonçalves, R.S.: Reliability-based optimization of flexible manipulators. Journal of Vibration Engineering & Technologies **11**(7) (2023)

[51] Lara-Molina, F.A., Dumur, D.: Robust multi-objective optimization of parallel manipulators. Meccanica **56**(11), 2843–2860 (2021)

[52] Xu, D.: Kinematic reliability and sensitivity analysis of the modified delta parallel mechanism. International Journal of Advanced Robotic Systems **15**(1), 1729881418759106 (2018)

[53] Chen, A., Kim, J., Lee, S., Kim, Y.: Stochastic multi-objective models for network design problem. Expert Systems with Applications **37**(2), 1608–1619 (2010)

[54] Cui, G., Zhang, H., Zhang, D., Xu, F.: Analysis of the kinematic accuracy reliability of a 3-dof parallel robot manipulator. International Journal of Advanced Robotic Systems **12**(2), 15 (2015)

[55] Zhan, Z., Zhang, X., Jian, Z., Zhang, H.: Error modelling and motion reliability analysis of a planar parallel manipulator with multiple uncertainties. Mechanism and Machine Theory **124**, 55–72 (2018)

[56] Zhang, D., Han, X.: Kinematic reliability analysis of robotic manipulator. Journal of Mechanical Design **142**(4), 044502 (2020)

[57] Zhang, X., Pandey, M.D., Zhang, Y.: Computationally efficient reliability analysis of mechanisms based on a multiplicative dimensional reduction method. Journal of Mechanical Design **136**(6), 061006 (2014)

[58] Wu, J., Zhang, D., Liu, J., Han, X.: A moment approach to positioning accuracy reliability analysis for industrial robots. IEEE Transactions on Reliability **69**(2), 699–714 (2019)

[59] Zhang, X., Pandey, M.D.: An efficient method for system reliability analysis of planar mechanisms. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science **227**(2), 373–386 (2013)

[60] Lara-Molina, F.A., Dumur, D.: Global performance criterion of robotic manipulator with clearances based on reliability. Journal of the Brazilian Society of Mechanical Sciences and Engineering **42**(12), 624 (2020)

[61] Geng, X., Wang, X., Wang, L., Wang, R.: Non-probabilistic time-dependent kinematic reliability assessment for function generation mechanisms with joint clearances. Mechanism and Machine Theory **104**, 202–221 (2016)

[62] Geng, X., Li, M., Liu, Y., Zheng, W., Zhao, Z.: Non-probabilistic kinematic reliability analysis of planar mechanisms with non-uniform revolute clearance joints. Mechanism and Machine Theory **140**, 413–433 (2019)

[63] Lara-Molina, F.A., Gonçalves, R.S.: Kinematic reliability of manipulators based on an interval approach. Robotics **13**(11), 155 (2024)

[64] Lara-Molina, F.A., Dumur, D.: A fuzzy approach for the kinematic reliability assessment of robotic manipulators. Robotica **39**(12), 2095–2109 (2021)

[65] Faes, M., Moens, D.: Recent trends in the modeling and quantification of non-probabilistic uncertainty. Archives of Computational Methods in Engineering **27**(3), 633–671 (2020)

[66] Kang, R., Zhang, Q., Zeng, Z., Zio, E., Li, X.: Measuring reliability under epistemic uncertainty: Review on non-probabilistic reliability metrics. Chinese

Journal of Aeronautics **29**(3), 571–579 (2016)

[67] Chang, Q., Zhou, C., Wei, P., Zhang, Y., Yue, Z.: A new non-probabilistic time-dependent reliability model for mechanisms with interval uncertainties. Reliability Engineering & System Safety **215**, 107771 (2021)

[68] Yang, C., Lu, W., Xia, Y.: Positioning accuracy analysis of industrial robots based on non-probabilistic time-dependent reliability. IEEE Transactions on Reliability **73**(1), 608–621 (2023)

[69] Tang, Z., Peng, J., Sun, J., Meng, X.: Non-probabilistic reliability analysis of robot accuracy under uncertain joint clearance. Machines **10**(10), 917 (2022)