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Advances in the use of Systems of Equalities and Inequalities to an analytical demonstration the Four-Color Theorem.

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Abstract

The Four-Color Theorem was first solved using computational methods in 1977, after more than 125 years of attempts. The only currently accepted approaches are those derived from the original paper by Appel and Haken. Although efforts to solve this problem have led to the development of new branches of mathematics, an analytical proof remains elusive. A simple internet search reveals hundreds of unrecognized attempts at analytical solutions proposed by numerous researchers.

This article presents a brief comment on all successes that are based in well stablished computational demonstrations and fails that are based in analytical approaches. The inexistence of an analytical demonstration and the incessant efforts in conquest it until now demonstrates that there is still a gap to be fulfilled. This article presents the improvement of an analytical demonstration proposed by this author in 2024. Differently from the current approaches, this author transformed the Four-Color Theorem into a system of equalities and inequalities once this type of system belongs to the fundamentals of algebra. Then, instead of using new tools, new concepts, new theories, new definitions, the proposal is to use only well stablished tolls in the demonstration.

In the previous article, the starting point was to reinterpret the problem as one of coloring “countries” distributed over a spherical surface. Spherical coordinates were employed to map the surface onto a plane. The concept of a *hyperline* was introduced and added to this plane, and a corresponding system of algebraic equalities and inequalities was established.

In the present article, these concepts are revisited. Additional considerations regarding the algebraic system are developed, and the map-coloring problem is reformulated as a problem of coloring segments of the corresponding *hyperline* of a map. The resulting system is solved, offering a simple, clear, and elegant demonstration of the Four-Color Theorem.

Introduction - A voyage through recognized and non-recognized demonstration of the Four-Color Theorem

The problem of the four colors arose in 1852 when Francis Guthrie observed that four colors seemed to be sufficient to color any map of England. After a long process his conjecture was sent to Arthur Cayley, who published the problem in the Philosophical Magazine in 1879, and then the problem gained international notoriety. This problem was maintained open until 1977 when Kenneth Appel and Wolfgang Haken published the first accepted proof that generates controversy due to the use of a computer-assisted proof in the demonstration. After then constant advances were done improving the same approach. In 1997, Neil Robertson, Daniel Sanders, Paul Seymour, and Robin Thomas presented a simplified version of Haken's proof. In 2008, Georges Gonthier and Benjamin Werner published a proof using a computer-assisted proof system called Coq, formalizing Robertson et al.'s proof. In 2012, Thomas Hales announced a project called Flyspeck using another computer-assisted proof system called HOL Light. The project was completed in 2014 and published in the Forum of Mathematics Pi in 2017. In 2016, Marijn Heule, Oliver Kullmann, and Victor Marek presented a proof of the Four-Color Theorem using a resolution-based problem-solving method based on Boolean satisfiability (SAT). They reduced the problem to 633 Boolean formulas, each representing an unavoidable configuration, and used a SAT solver to verify their colorability that was published in the Journal of Automated Reasoning in 2017."

By other side, searching aleatorily at internet there are a lot of titles involving the words "Four Color". A little sample of collected articles is presented (Nguyen, Brierly, Bhupinder, Voloshin, Yegnanarayanan, Tshwane and Tosuni). Reading these references, it can be observed that they have in common lots of distinct concepts, complex definitions, dozens of technical terms and hard demonstrations that seems to be the fingerprint of the analytical approach. The reader is invited to read them to confirm this observation. Other articles visited and not reported have the same characteristics. They are an incontestable proof of the erudition of the authors around the subject but the inexistence of a recognized analytical proof of the Four-Color Theorem is an uncomfortable situation.

Faced to this situation a question arises. Is it possible to present a less technical and more intuitive approach to obtain an analytical demonstration of the Four-Color Theorem? This is the goal of this article.

Rescuing the Previous Article

In the previous article of Jansen, the Earth's map was represented as a three-dimensional sphere using the coordinates (R, θ, φ). Subsequently, the radius was set as R = 1, converting the generic spherical surface into a unitary spherical surface. Since R is constant, the representation is reduced to a function of the angular coordinates (θ, φ).

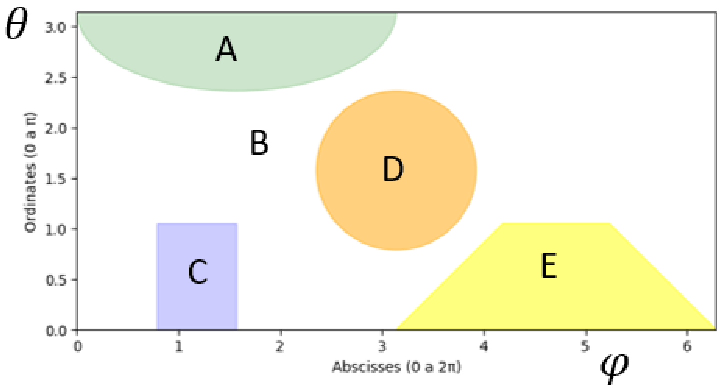

Fig 1-) Example of a (θ, φ) plane representation of a MAP

In the same article, the (θ, φ) plane is juxtaposed an integer number **R-1** times along the φ-axis, as illustrated in Fig. 2. A line, referred to as the *hyperline*, is drawn passing through the **R** copies of the map, connecting the initial point (0, 0) to the final point (R·2·π, π). The value of **R** is chosen such that the hyperline intersects all countries at least once. As **R** approaches infinity, the hyperline intersects every country at least once, and each country contains at least one segment of the hyperline.

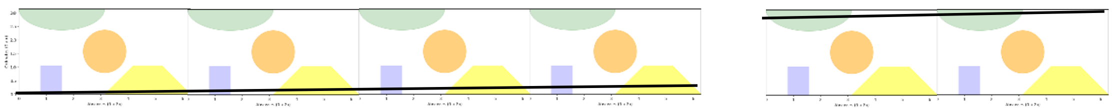

Fig 2-) Juxtaposition of the R copies of (θ, φ) planes and the correspondent hyperline passing through the R copies

The $\mathbf{R-1}$ copies can be superimposed onto the first copy of the (θ, φ) plane, resulting in the appearance of segmented lines denoted as $\boldsymbol{S}^{\boldsymbol{k}}_{\boldsymbol{l(k,\ \varphi)}}$. Here k represents the k-th line and $l_{(k,\boldsymbol{\varphi})}$represents the l-th segment of the k-th line at the φ abscissa as presented in Fig 3.

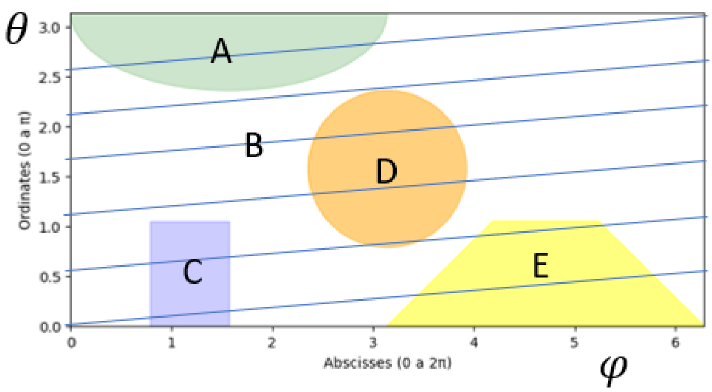

Fig 3-) The superposed (R-1)*(θ, φ) copies over the first copy.

New Developments Based on the Hyperline Concept

According to Fig. 3, a set of linear functions $\theta(k,\varphi) = \theta_k + \left(\frac{1}{2R}\right) * \varphi = \left(\frac{k}{R}\right)\pi + \left(\frac{1}{2R}\right)\varphi$ describes the result of the superposition of the hyperline onto the first copy. The functions $\theta(k,\varphi)$ are continuous with respect to φ a and discontinuous with respect to k. Each function indexed by k is subdivided into a sequence of segments defined by the intersection points between the k-lines and the boundaries of the countries. These intersection points are, at this stage, assumed not to belong to the problem and will not be considered for now (the original formulation does not treat the borders as part of the problem. This aspect will be addressed at the end of the text).

At the k-th line, each point $\theta(k,\varphi)$ at abscissa φ has a neighbor $\varphi - \varepsilon$ at left and $\varphi + \varepsilon$ at right (these neighbors are not defined when φ corresponds to a cutting point). The corresponding levels are given by $\theta_{\varphi-\varepsilon} = \left(\frac{k}{R}\right)\pi + \left(\frac{1}{2R}\right)(\varphi - \varepsilon)$ and $\theta_{\varphi+\varepsilon} = \left(\frac{k}{R}\right)\pi + \left(\frac{1}{2R}\right)(\varphi + \varepsilon)$). Thus, for each point $\theta(,\varphi)$ contained in a segment $\boldsymbol{S^k_{l(k,\varphi)}}$ there are four associated neighbors: one to the left, $S^k_{l(\varphi-\varepsilon)}$, one at right $S^k_{l(\varphi+\varepsilon)}$, and at the selected abscissa φ one neighbor at $S^{k-1}_{l(\varphi)}$ and other at $S^{k+1}_{l(\varphi)}$. It is important to note that the l-th segment index at (k-1)-th, (k)-th and (k+1)-th are independent each other. To simplify the nomenclature for discrete segments in φ, $l(\varphi)$ can be written as l_1, l_2, l_3, etc., or simply as 1, 2, 3, and so on. Colors can be associated with the segments as parameters, under the assumption that if two segments are neighbors, they must be assigned different colors. Using this discrete segment notation and enumeration, it can be observed in Fig. 4 that $S^3_1$is a continuation of S^1_2 due to the absence of a border point between
them.

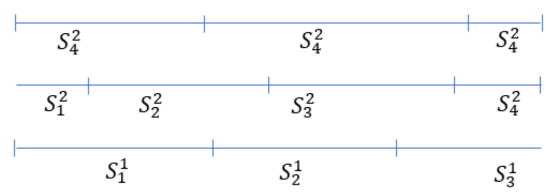

Fig 4-) Example of enumeration of segments. Repair that S^1_3 is continued at S^2_1 .

From Fig. 4, by distinguishing neighboring segments based on their border points and associated k-lines, a system of equalities and inequalities can be formulated, as presented in Fig. 5.

1	2	3	4
1.1) $S^2_1 \neq S^1_1$	2.1) $S^2_2 \neq S^1_1$	3.1) $S^2_3 \neq S^1_2$	4.1) $S^2_4 \neq S^1_3$
1.2) $\boldsymbol{S^2_1 = S^1_3}$	2.2) $S^2_2 \neq S^1_2$	3.2) $S^2_3 \neq S^1_3$	4.2) $S^2_4 \neq S^2_3$
1.3) $S^2_1 \neq S^2_2$	2.3) $S^2_2 \neq S^2_1$	3.3) $S^2_3 \neq S^2_2$	4.3) $S^2_4 \neq S^3_1$
1.4) $S^2_1 \neq S^3_1$	2.4) $S^2_2 \neq S^2_3$	3.4) $S^2_3 \neq S^2_4$	4.4) $S^2_4 \neq S^3_2$
	2.5) $S^2_2 \neq S^3_1$	3.5) $S^2_3 \neq S^3_2$	4.5) $S^2_4 \neq S^3_3$
	2.6) $S^2_2 \neq S^3_2$		

Figure 5-) Inequalities System for the segments of line 2.

Neighboring is a reflexive property (if segment A is a neighbor of segment B, then segment B is also a neighbor of segment A). Therefore, each equality or inequality is written twice in the complete system. Consequently, the set of constraints presented in Fig. 5 can be reduced.

It is noteworthy that the number of constraints per segment, as well as per (θ, φ) can always be reduced to four. If the same rules illustrated in Fig. 5 are applied to the segments S_1^2, S_2^3, S_3^1 and S_3^3 the resulting redundancies allow for the elimination of constraints R2.2, R2.6, R3.2 and R4.5 as shown in Fig. 5. The simplified system is then rewritten in Fig. 6.

1.1) $S_1^2 \neq S_1^1$	2.1) $S_2^2 \neq S_1^1$	3.1) $S_3^2 \neq S_2^1$	4.1) $S_4^2 \neq S_3^1$
1.2) $\boldsymbol{S_1^2 = S_3^1}$	2.2) $S_2^2 \neq S_1^2$	3.2) $S_3^2 \neq S_2^2$	4.2) $S_4^2 \neq S_3^2$
1.3) $S_1^2 \neq S_2^2$	2.3) $S_2^2 \neq S_3^2$	3.3) $S_3^2 \neq S_4^2$	4.3) $S_4^2 \neq S_1^3$
1.4) $S_1^2 \neq S_1^3$	2.4) $S_2^2 \neq S_1^3$	3.4) $S_3^2 \neq S_2^3$	4.4) $S_4^2 \neq S_2^3$

Fig 6-) Example of the Selection rules applied to the inequalities System of Fig 5
Then every segment is subjected to 4 (four) restrictions.

From this point forward, the notations $S_{l(\varphi)}^k$ or $S_{l_i}^k$ or S_i^k will be used interchangeably. For the hyperline, the **complete set of possible constraints** associated with each segment is summarized in Table 1.

R1.1	$S_{l(\varphi)}^k = S_{l(\varphi-\varepsilon)}^k$;	$S_{l(\varphi)}^k = S_{l(\varphi+\varepsilon)}^k$;	$S_{l(\varphi)}^k = S_{l(\varphi)}^{k-1}$;	$S_{l(\varphi)}^k = S_{l(\varphi)}^{k+1}$
R2.1	$S_{l(\varphi)}^k = S_{l(\varphi-\varepsilon)}^k$;	$S_{l(\varphi)}^k = S_{l(\varphi+\varepsilon)}^k$;	$S_{l(\varphi)}^k = S_{l(\varphi)}^{k-1}$;	$S_{l(\varphi)}^k \neq S_{l(\varphi)}^{k+1}$
R2.2	$S_{l(\varphi)}^k = S_{l(\varphi-\varepsilon)}^k$;	$S_{l(\varphi)}^k = S_{l(\varphi+\varepsilon)}^k$;	$S_{l(\varphi)}^k \neq S_{l(\varphi)}^{k-1}$;	$S_{l(\varphi)}^k = S_{l(\varphi)}^{k+1}$
R2.3	$S_{l(\varphi)}^k = S_{l(\varphi-\varepsilon)}^k$;	$S_{l(\varphi)}^k = S_{l(\varphi+\varepsilon)}^k$;	$S_{l(\varphi)}^k \neq S_{l*}^{k-1}$;	$S_{l(\varphi)}^k \neq S_{l(\varphi)}^{k+1}$
R2.4	$S_{l(\varphi)}^k = S_{l(\varphi-\varepsilon)}^k$;	$S_{l(\varphi)}^k \neq S_{l(\varphi+\varepsilon)}^k$;	$S_{l(\varphi)}^k = S_{l(\varphi)}^{k-1}$;	$S_{l(\varphi)}^k = S_{l(\varphi)}^{k+1}$
R3.1	$S_{l(\varphi)}^k = S_{l(\varphi-\varepsilon)}^k$;	$S_{l(\varphi)}^k \neq S_{l(\varphi+\varepsilon)}^k$;	$S_{l(\varphi)}^k = S_{l(\varphi)}^{k-1}$;	$S_{l(\varphi)}^k \neq S_{l(\varphi)}^{k+1}$
R3.2	$S_{l(\varphi)}^k = S_{l(\varphi-\varepsilon)}^k$;	$S_{l(\varphi)}^k \neq S_{l(\varphi+\varepsilon)}^k$;	$S_{l(\varphi)}^k \neq S_{l*}^{k-1}$;	$S_{l(\varphi)}^k = S_{l(\varphi)}^{k+1}$
R3.3	$S_{l(\varphi)}^k = S_{l(\varphi-\varepsilon)}^k$;	$S_{l(\varphi)}^k \neq S_{l(\varphi+\varepsilon)}^k$;	$S_{l(\varphi)}^k \neq S_{l(\varphi)}^{k-1}$;	$S_{l(\varphi)}^k \neq S_{l(\varphi)}^{k+1}$
R3.4	$S_{l(\varphi)}^k \neq S_{l(\varphi-\varepsilon)}^k$;	$S_{l(\varphi)}^k = S_{l(\varphi+\varepsilon)}^k$;	$S_{l(\varphi)}^k = S_{l(\varphi)}^{k-1}$;	$S_{l(\varphi)}^k = S_{l(\varphi)}^{k+1}$
R3.5	$S_{l(\varphi)}^k \neq S_{l(\varphi-\varepsilon)}^k$;	$S_{l(\varphi)}^k = S_{l(\varphi+\varepsilon)}^k$;	$S_{l(\varphi)}^k = S_{l(\varphi)}^{k-1}$;	$S_{l(\varphi)}^k \neq S_{l(\varphi)}^{k+1}$
R3.6	$S_{l(\varphi)}^k \neq S_{l(\varphi-\varepsilon)}^k$;	$S_{l(\varphi)}^k = S_{l(\varphi+\varepsilon)}^k$;	$S_{l(\varphi)}^k \neq S_{l*}^{k-1}$;	$S_{l(\varphi)}^k = S_{l**}^{k+1}$
R4.1	$S_{l(\varphi)}^k \neq S_{l(\varphi-\varepsilon)}^k$;	$S_{l(\varphi)}^k = S_{l(\varphi+\varepsilon)}^k$;	$S_{l(\varphi)}^k \neq S_{l(\varphi)}^{k-1}$;	$S_{l(\varphi)}^k \neq S_{l(\varphi)}^{k+1}$
R4.2	$S_{l(\varphi)}^k \neq S_{l(\varphi-\varepsilon)}^k$;	$S_{l(\varphi)}^k \neq S_{l(\varphi+\varepsilon)}^k$;	$S_{l(\varphi)}^k = S_{l(\varphi)}^{k-1}$;	$S_{l(\varphi)}^k = S_{l(\varphi)}^{k+1}$

R4.3	$S^k_{l(\varphi)} \neq S^k_{l(\varphi-\varepsilon)}$;	$S^k_{l(\varphi)} \neq S^k_{l(\varphi+\varepsilon)}$;	$S^k_{l(\varphi)} = S^{k-1}_{l(\varphi)}$;	$S^k_{l(\varphi)} \neq S^{k+1}_{l(\varphi)}$
R4.4	$S^k_{l(\varphi)} \neq S^k_{l(\varphi-\varepsilon)}$;	$S^k_{l(\varphi)} \neq S^k_{l(\varphi+\varepsilon)}$;	$S^k_{l(\varphi)} \neq S^{k-1}_{l(\varphi)}$;	$S^k_{l(\varphi)} = S^{k+1}_{l(\varphi)}$
R5.1	$S^k_{l(\varphi)} \neq S^k_{l(\varphi-\varepsilon)}$;	$S^k_{l(\varphi)} \neq S^k_{l(\varphi+\varepsilon)}$;	$S^k_{l(\varphi)} \neq S^{k-1}_{l(\varphi)}$;	$S^k_{l(\varphi)} \neq S^{k+1}_{l(\varphi)}$

Table 1: All possible logical relations (colors of) between neighboring points $S^k_{l(\varphi)}$ ($S^k_{l(\varphi-\varepsilon)}$, $S^k_{l(\varphi+\varepsilon)}$, $S^{k-1}_{l(\varphi)}$, $S^{k+1}_{l(\varphi)}$) of the hyperline.

As previously stated, the above relationships between points can be extended to segments, and the same constraints can be written by replacing the continuous indices $l(\varphi-\varepsilon)$, $l(\varphi)$, $l(\varphi+\varepsilon)$ by $l-1$, by l and by $l+1$ respectively.
Since the hyperline can perform a complete cycle, it can be asserted that each segment is involved in five equalities or inequalities: one with index k-1 and segment l*, tree at index k, and segments l-1, l, and l+1 and one at index k+1 and segment l**.

When the hyperline concept is applied to a MAP, by definition, a-) sequential segments along the same line belongs to different countries; b-) Sequential segments belonging to adjacent lines may or may not belong to different countries (as illustrated by segments S^1_3 and S^2_1 of Fig 4). Remark: Rule a-) can be disregarded without any interference in the solutions of the system that is, a segment S^l_k can be subdivided into equivalent n sub segments S^l_{k1} , S^l_{k2} ,… S^l_{kn} as long as $S^l_{k1} = S^l_{k2} = \cdots S^l_{kn}$ is considered. This observation is considered from now on for the sake of brevity in the deductions that follow.
By examining Table 1, it can be observed that a maximum of five colors is required to satisfy all constraints.

Deepening and stressing the arguments.

To ensure that all neighboring relationships are represented in the system, the number of replications is increased by a factor M, guaranteeing that at least two segments of the **Hyperline** are contained within each country. (Here, the **Hyperline** is re-defined as the line connecting the points (0,0) and (M*R*2* π, π).

As in Fig. 3, the (M·R − 1) copies can be superimposed onto the first copy, as illustrated in Fig. 7.

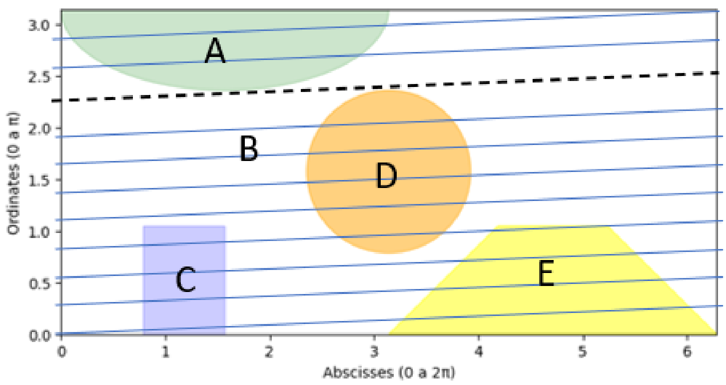

Fig 7 -) The superposed (M*R -1)(θ, φ) copies of the planes over the first copy.

As **M·R** is chosen in such a way that at least two or more segments are contained within each country, the restrictions $S^{k}_{l(\varphi)} \neq S^{k-1}_{l(\varphi)}$ and $S^{k}_{l(\varphi)} \neq S^{k+1}_{l(\varphi)}$ will never occur simultaneously. Therefore, the restrictions R2.3, R3.3, R4.1, and R5.1 will never occur in MAPs as they are currently conceptualized.

Reinforcing this observation, in order to represent a valid MAP, the segment $\boldsymbol{S^{k}_{l}}$ must satisfy only one of the following conditions:

- $\boldsymbol{S^{k}_{l}}$ is equal to $\boldsymbol{S^{k+1}_{l**}}$ **and** equal to $\boldsymbol{S^{k-1}_{l*}}$
- $\boldsymbol{S^{k}_{l}}$ is equal to $\boldsymbol{S^{k+1}_{l**}}$ **and** different from $\boldsymbol{S^{k-1}_{l*}}$**,** or
- $\boldsymbol{S^{k}_{l}}$ is equal to $\boldsymbol{S^{k-1}_{l*}}$ **and** different of $\boldsymbol{S^{k+1}_{l**}}$.

Therefore, since restrictions **R2.3, R3.3, R4.1 and R5.1** do not comply with these conditions, they are not MAPs and are then designated as **Non-MAPs**.

To conclude the demonstration, instead of coloring MAPs, the segments of the **Hyperline** will now be colored using different and independent colors.

It can be stated that:

- If **six** colors (1, 2, 3, 4, 5, 6) are independently assigned to the segments of the **Hyperline**, then the **Hyperline** has $6^{\Sigma \text{segments}}$ possible states (each state is in bijective correspondence with a number between 111...1 and 666...6, representing color sequences of the **Hyperline** segments). Among these, some states represent valid MAPs, while others correspond to Non-MAPs (those represented by restrictions such as **R2.3, R3.3, R4.1 and R5.1**)
- If **five** colors are used, then the **Hyperline** has $5^{\Sigma \text{segments}}$ possible states, part of which represent valid MAPs and part represent Non-MAPs.
- If **four** colors are used, then the **Hyperline** has $4^{\Sigma \text{segments}}$ possible states, again comprising both MAPs and Non-MAPs.
- If **three** colors are used, then the **Hyperline** has $3^{\Sigma \text{segments}}$ possible states, and although some MAPs may still be representable those ones satisfying restrictions **R4.3 and R4.4 that are viable MAPs cannot be represented.**

Therefore, four colors are the minimum number of colors necessary to color any two-dimensional map.

An Intersection Between Complex Functions and the colors of the Hyperline.

Suppose that, one of the following complex equations is associated for each segment of the **Hyperline**:

- $f_1 = +(\theta + \left(\frac{1}{2R}\right)\varphi) = 0,\ \varphi_{begin} \leq \varphi \leq \varphi_{end},\ 0 \leq \varphi \leq 2MR\pi$
- $f_2 = -(\theta + \left(\frac{1}{2R}\right)\varphi) = 0,\ \varphi_{begin} \leq \varphi \leq \varphi_{end},\ 0 \leq \varphi \leq 2MR\pi$

- $f_3 = +i(\theta + \left(\frac{1}{2R}\right)\varphi) = 0, \varphi_{begin} \leq \varphi \leq \varphi_{end}, 0 \leq \varphi \leq 2MR\pi$
- $f_4 = -i(\theta + \left(\frac{1}{2R}\right)\varphi) = 0, \varphi_{begin} \leq \varphi \leq \varphi_{end}, 0 \leq \varphi \leq 2MR\pi$

Each of these linear functions f_1, f_2, f_3 and f_4 behaves as a distinct "color"(they divide the plane in non-superimposable parts). This interpretation provides a natural solution to the issue of frontier points, which had not previously been addressed.

In a prior article of Jansen, the author encountered a similar complex equation structure through a different line of reasoning. On that occasion, it was suggested that the decomposition of a real-valued equation into four complex-valued equations may be worth further exploration—at the very least as a theoretical curiosity. Such a structure may indeed have practical applications.

Conclusion

Starting from standard mathematical assumptions, the Four-Color Theorem is demonstrable. The only hypothesis adopted in this work is that infinitesimal deformations can be applied to the frontiers in order to circumvent anomalies—such as those that occur when for example the **Hyperline** intersects multivalued points *P* or continuous sections of frontiers *L*, as illustrated in Fig. 8.

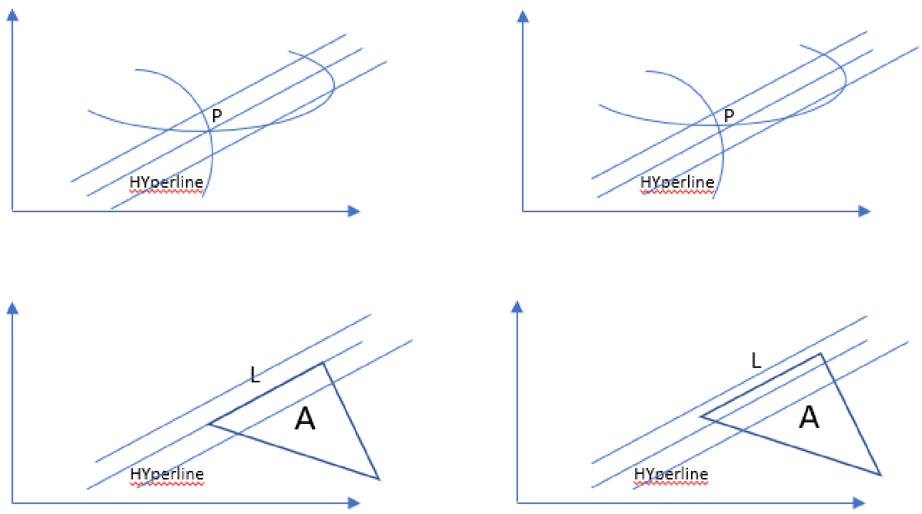

Fig 8-) Illustration of the intersection between an **Hyperline** and a multivalued point **P** or at a continuous piece **L** frontier at left side of the figure and how they can be displaced by infinitesimal deformations at right side of the figure.

Conclusion (complement)

Once the mechanism of infinitesimal deformations is accepted, the subsequent text presents a logical sequence of algebraic steps governed by well-established mathematical rules.

This new development yields two noteworthy outcomes:
a) the reduction of the number of restrictions to four per segment, a consequence of the

reflexive property of neighboring relations; and
b) the natural elimination of restrictions R2.3, R3.3, R4.1, and R5.1 as valid MAPs representation, solely by assuming that each country contains more than one segment of the **Hyperline**.

A unified demonstration encompassing (from one to) four, five, six, or more colors is derived within the same analytical framework.

This article is not a treatise on the subject and of course may receive contributions from others but, as manifested in previous articles, the author hopes that the ideas presented herein signalizes the emergence of a robust analytical proof of the Four-Color Theorem. The author also acknowledges the support of Mrs. Natália C. J. Casanova and Dr. Juliano Alves Bonacim. Let's Rock and Roll!

Conflicts of interest

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References

Appel, K., & Haken, W. (1977). Every Planar Map is Four Colorable. Part I: Discharging. Illinois Journal of Mathematics, 21(3), 429-490.
Appel, K., & Haken, W. (1977). Every Planar Map is Four Colorable. Part II: Reducibility. Illinois Journal of Mathematics, 21(4), 491-567.
Robertson, N., Sanders, D., Seymour, P., & Thomas, R. (1997). The Four-Colour Theorem. Journal of Combinatorial Theory, Series B, 70(1), 2-44.
Gonthier, G., & Werner, B. (2008). The Four-Colour Theorem: Engineering of a Formal Proof. In Computer Mathematics (pp. 333-333). Springer Berlin Heidelberg.
Hales, T., Adams, M., Bauer, G., Dang, T., Harrison, J., Hoang, L., ... & Zumkeller, R. (2017). A Formal Proof of the Kepler Conjecture. Forum of Mathematics Pi, 5.

Heule, M., Kullmann, O., & Marek, V. (2017). Solving and Verifying the Boolean Pythagorean Triples Problem via Cube-and-Conquer. In Journal of Automated Reasoning (pp. 1-37). Springer International Publishing."
Nguyen Van Quang. A PROOF OF THE FOUR-COLOR THEOREM BY INDUCTION. Vietnan 01-2016 (independently published)
Joseph Edward Brierly. Generalized 2N Color Theorem. J Phy Opt Sci, 2020, Volume 2(4) 1-4
Bhupinder Singh Anand. A pictorial proof of the Four Colour Theorem. https://arxiv.org/pdf/2110.09718
Vitaly Voloshin, Coloring theory: history, results and open problems. lecture at the annual meeting of AACTM. Troy University, USA. February 28, 2009. (independently published)
V. Yegnanarayanan. On Vertex Coloring of Graphs. International Journal of Mathematical Analysis Vol. 9, 2015, no. 17, 857 – 868.
Kok Tshwane. On New Thue Colouring Concepts of Certain Graphs Johan Metropolitan Police Department City of Tshwane, Republic of South Afric
Besjana Tosuni. Graph Coloring Problems in Modern Computer Science. European Journal of Interdisciplinary Studies May-August 2015 Volume 1, Issue 2, 87-95
Jansen, J. U. (2024). Use of Equalities and Inequalities System to demonstrate the Four-Color Theorem. In *SciELO Preprints*. https://doi.org/10.1590/SciELOPreprints.8012
Jansen, J. (2021). Proposta de Solução Algébrica para o Teorema das 4 Cores. In *SciELO Preprints*. https://doi.org/10.1590/SciELOPreprints.3156